

# NAG Library Function Document

## nag\_dgetrf (f07adc)

### 1 Purpose

nag\_dgetrf (f07adc) computes the  $LU$  factorization of a real  $m$  by  $n$  matrix.

### 2 Specification

```
#include <nag.h>
#include <nagf07.h>

void nag_dgetrf (Nag_OrderType order, Integer m, Integer n, double a[],
                Integer pda, Integer ipiv[], NagError *fail)
```

### 3 Description

nag\_dgetrf (f07adc) forms the  $LU$  factorization of a real  $m$  by  $n$  matrix  $A$  as  $A = PLU$ , where  $P$  is a permutation matrix,  $L$  is lower triangular with unit diagonal elements (lower trapezoidal if  $m > n$ ) and  $U$  is upper triangular (upper trapezoidal if  $m < n$ ). Usually  $A$  is square ( $m = n$ ), and both  $L$  and  $U$  are triangular. The function uses partial pivoting, with row interchanges.

### 4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

### 5 Arguments

- 1: **order** – Nag\_OrderType *Input*  
*On entry:* the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag\_RowMajor. See Section 3.2.1.3 in the Essential Introduction for a more detailed explanation of the use of this argument.  
*Constraint:* **order** = Nag\_RowMajor or Nag\_ColMajor.
- 2: **m** – Integer *Input*  
*On entry:*  $m$ , the number of rows of the matrix  $A$ .  
*Constraint:*  $m \geq 0$ .
- 3: **n** – Integer *Input*  
*On entry:*  $n$ , the number of columns of the matrix  $A$ .  
*Constraint:*  $n \geq 0$ .
- 4: **a**[*dim*] – double *Input/Output*  
**Note:** the dimension, *dim*, of the array **a** must be at least  
 $\max(1, \mathbf{pda} \times \mathbf{n})$  when **order** = Nag\_ColMajor;  
 $\max(1, \mathbf{m} \times \mathbf{pda})$  when **order** = Nag\_RowMajor.

The  $(i, j)$ th element of the matrix  $A$  is stored in

$$\begin{aligned} & \mathbf{a}[(j-1) \times \mathbf{pda} + i - 1] \text{ when } \mathbf{order} = \text{Nag\_ColMajor}; \\ & \mathbf{a}[(i-1) \times \mathbf{pda} + j - 1] \text{ when } \mathbf{order} = \text{Nag\_RowMajor}. \end{aligned}$$

*On entry:* the  $m$  by  $n$  matrix  $A$ .

*On exit:* the factors  $L$  and  $U$  from the factorization  $A = PLU$ ; the unit diagonal elements of  $L$  are not stored.

5: **pda** – Integer *Input*

*On entry:* the stride separating row or column elements (depending on the value of **order**) in the array **a**.

*Constraints:*

$$\begin{aligned} & \text{if } \mathbf{order} = \text{Nag\_ColMajor}, \mathbf{pda} \geq \max(1, \mathbf{m}); \\ & \text{if } \mathbf{order} = \text{Nag\_RowMajor}, \mathbf{pda} \geq \max(1, \mathbf{n}). \end{aligned}$$

6: **ipiv**[**min**(**m**, **n**)] – Integer *Output*

*On exit:* the pivot indices that define the permutation matrix. At the  $i$ th step, if **ipiv**[ $i-1$ ]  $> i$  then row  $i$  of the matrix  $A$  was interchanged with row **ipiv**[ $i-1$ ], for  $i = 1, 2, \dots, \min(m, n)$ . **ipiv**[ $i-1$ ]  $\leq i$  indicates that, at the  $i$ th step, a row interchange was not required.

7: **fail** – NagError \* *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

### NE\_BAD\_PARAM

On entry, argument  $\langle \text{value} \rangle$  had an illegal value.

### NE\_INT

On entry, **m** =  $\langle \text{value} \rangle$ .

Constraint: **m**  $\geq 0$ .

On entry, **n** =  $\langle \text{value} \rangle$ .

Constraint: **n**  $\geq 0$ .

On entry, **pda** =  $\langle \text{value} \rangle$ .

Constraint: **pda**  $> 0$ .

### NE\_INT\_2

On entry, **pda** =  $\langle \text{value} \rangle$  and **m** =  $\langle \text{value} \rangle$ .

Constraint: **pda**  $\geq \max(1, \mathbf{m})$ .

On entry, **pda** =  $\langle \text{value} \rangle$  and **n** =  $\langle \text{value} \rangle$ .

Constraint: **pda**  $\geq \max(1, \mathbf{n})$ .

### NE\_INTERNAL\_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

**NE\_SINGULAR**

$U(\langle value \rangle, \langle value \rangle)$  is exactly zero. The factorization has been completed, but the factor  $U$  is exactly singular, and division by zero will occur if it is used to solve a system of equations.

**7 Accuracy**

The computed factors  $L$  and  $U$  are the exact factors of a perturbed matrix  $A + E$ , where

$$|E| \leq c(\min(m, n))\epsilon P|L||U|,$$

$c(n)$  is a modest linear function of  $n$ , and  $\epsilon$  is the *machine precision*.

**8 Parallelism and Performance**

nag\_dgetrf (f07adc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

nag\_dgetrf (f07adc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the Users' Note for your implementation for any additional implementation-specific information.

**9 Further Comments**

The total number of floating-point operations is approximately  $\frac{2}{3}n^3$  if  $m = n$  (the usual case),  $\frac{1}{3}n^2(3m - n)$  if  $m > n$  and  $\frac{1}{3}m^2(3n - m)$  if  $m < n$ .

A call to this function with  $m = n$  may be followed by calls to the functions:

nag\_dgetrs (f07aec) to solve  $AX = B$  or  $A^T X = B$ ;

nag\_dgecon (f07agc) to estimate the condition number of  $A$ ;

nag\_dgetri (f07ajc) to compute the inverse of  $A$ .

The complex analogue of this function is nag\_zgetrf (f07arc).

**10 Example**

This example computes the  $LU$  factorization of the matrix  $A$ , where

$$A = \begin{pmatrix} 1.80 & 2.88 & 2.05 & -0.89 \\ 5.25 & -2.95 & -0.95 & -3.80 \\ 1.58 & -2.69 & -2.90 & -1.04 \\ -1.11 & -0.66 & -0.59 & 0.80 \end{pmatrix}.$$

**10.1 Program Text**

```
/* nag_dgetrf (f07adc) Example Program.
 *
 * Copyright 2001 Numerical Algorithms Group.
 *
 * Mark 7, 2001.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf07.h>
#include <nagx04.h>

int main(void)
```

```

{
    /* Scalars */
    Integer      i, ipiv_len, j, m, n, pda;
    Integer      exit_status = 0;
    NagError     fail;
    Nag_OrderType order;

    /* Arrays */
    double       *a = 0;
    Integer      *ipiv = 0;

#ifdef NAG_COLUMN_MAJOR
#define A(I, J) a[(J-1)*pda + I - 1]
    order = Nag_ColMajor;
#else
#define A(I, J) a[(I-1)*pda + J - 1]
    order = Nag_RowMajor;
#endif

    INIT_FAIL(fail);

    printf("nag_dgetrf (f07adc) Example Program Results\n\n");

    /* Skip heading in data file */
    scanf("%*[\n] ");
    scanf("%ld%ld%*[\n] ", &m, &n);
    ipiv_len = MIN(m, n);
#ifdef NAG_COLUMN_MAJOR
    pda = m;
#else
    pda = n;
#endif

    /* Allocate memory */
    if (!(a = NAG_ALLOC(m * n, double)) ||
        !(ipiv = NAG_ALLOC(ipiv_len, Integer)))
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }

    /* Read A from data file */
    for (i = 1; i <= m; ++i)
    {
        for (j = 1; j <= n; ++j)
            scanf("%lf", &A(i, j));
    }
    scanf("%*[\n] ");

    /* Factorize A */
    /* nag_dgetrf (f07adc).
    * LU factorization of real m by n matrix
    */
    nag_dgetrf(order, m, n, a, pda, ipiv, &fail);
    if (fail.code != NE_NOERROR)
    {
        printf("Error from nag_dgetrf (f07adc).\n%s\n", fail.message);
        exit_status = 1;
        goto END;
    }

    /* Print details of factorization */
    /* nag_gen_real_mat_print (x04cac).
    * Print real general matrix (easy-to-use)
    */
    fflush(stdout);
    nag_gen_real_mat_print(order, Nag_GeneralMatrix, Nag_NonUnitDiag, m, n, a,
                           pda, "Details of factorization", 0, &fail);
    if (fail.code != NE_NOERROR)
    {

```

```

        printf("Error from nag_gen_real_mat_print (x04cac).\n%s\n",
               fail.message);
        exit_status = 1;
        goto END;
    }

    /* Print pivot indices */
    printf("\nipiv\n");
    for (i = 1; i <= MIN(m, n); ++i)
        printf("%6ld%s", ipiv[i-1], i%7 == 0?"\n":"" );
    printf("\n");

END:
    NAG_FREE(a);
    NAG_FREE(ipiv);
    return exit_status;
}

```

## 10.2 Program Data

```

nag_dgetrf (f07adc) Example Program Data
  4  4                               :Values of M and N
  1.80  2.88  2.05 -0.89
  5.25 -2.95 -0.95 -3.80
  1.58 -2.69 -2.90 -1.04
 -1.11 -0.66 -0.59  0.80   :End of matrix A

```

## 10.3 Program Results

```

nag_dgetrf (f07adc) Example Program Results

Details of factorization
      1          2          3          4
1      5.2500   -2.9500   -0.9500   -3.8000
2      0.3429    3.8914    2.3757    0.4129
3      0.3010   -0.4631   -1.5139    0.2948
4     -0.2114   -0.3299    0.0047    0.1314

ipiv
  2      2      3      4

```

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