NAG Library Function Document

nag_1d_cheb_intg (e02ajc)

1 Purpose

nag_1d_cheb_intg (e02ajc) determines the coefficients in the Chebyshev series representation of the indefinite integral of a polynomial given in Chebyshev series form.

2 Specification

```
#include <nag.h>
#include <nage02.h>
```

```
void nag_1d_cheb_intg (Integer n, double xmin, double xmax, const double a[],
     Integer ia1, double qatm1, double aintc[], Integer iaint1,
     NagError *fail)
```

3 Description

nag_1d_cheb_intg (e02ajc) forms the polynomial which is the indefinite integral of a given polynomial. Both the original polynomial and its integral are represented in Chebyshev series form. If supplied with the coefficients a_i, for $i = 0, 1, \dots, n$, of a polynomial $p(x)$ of degree n, where

$$p(x) = \tfrac{1}{2}a_0 + a_1 T_1(\bar{x}) + \cdots + a_n T_n(\bar{x}),$$

the function returns the coefficients a'_i, for $i = 0, 1, \dots, n+1$, of the polynomial $q(x)$ of degree $n+1$, where

$$q(x) = \tfrac{1}{2}a'_0 + a'_1 T_1(\bar{x}) + \cdots + a'_{n+1} T_{n+1}(\bar{x}),$$

and

$$q(x) = \int p(x)dx.$$

Here $T_j(\bar{x})$ denotes the Chebyshev polynomial of the first kind of degree j with argument $\bar{x}$. It is assumed that the normalized variable $\bar{x}$ in the interval $[-1, +1]$ was obtained from your original variable x in the interval $[x_{\min}, x_{\max}]$ by the linear transformation

$$\bar{x} = \frac{2x - (x_{\max} + x_{\min})}{x_{\max} - x_{\min}}$$

and that you require the integral to be with respect to the variable x. If the integral with respect to $\bar{x}$ is required, set $x_{\max} = 1$ and $x_{\min} = -1$.

Values of the integral can subsequently be computed, from the coefficients obtained, by using nag_1d_cheb_eval2 (e02akc).

The method employed is that of Chebyshev series (see Chapter 8 of Modern Computing Methods (1961)), modified for integrating with respect to x. Initially taking $a_{n+1} = a_{n+2} = 0$, the function forms successively

$$a'_i = \frac{a_{i-1} - a_{i+1}}{2i} \times \frac{x_{\max} - x_{\min}}{2}, \quad i = n+1, n, \dots, 1.$$

The constant coefficient a'_0 is chosen so that $q(x)$ is equal to a specified value, **qatm1**, at the lower end point of the interval on which it is defined, i.e., $\bar{x} = -1$, which corresponds to $x = x_{\min}$.

4 References

Modern Computing Methods (1961) Chebyshev-series *NPL Notes on Applied Science* **16** (2nd Edition) HMSO

5 Arguments

1: **n** – Integer *Input*

On entry: n, the degree of the given polynomial $p(x)$.

Constraint: **n** ≥ 0.

2: **xmin** – double *Input*

3: **xmax** – double *Input*

On entry: the lower and upper end points respectively of the interval $[x_{\text{min}}, x_{\text{max}}]$. The Chebyshev series representation is in terms of the normalized variable $\bar{x}$, where

$$\bar{x} = \frac{2x - (x_{\text{max}} + x_{\text{min}})}{x_{\text{max}} - x_{\text{min}}}.$$

Constraint: **xmax** $>$ **xmin**.

4: **a**[*dim*] – const double *Input*

Note: the dimension, *dim*, of the array **a** must be at least $(1 + (\mathbf{n} + 1 - 1) \times \mathbf{ia1})$.

On entry: the Chebyshev coefficients of the polynomial $p(x)$. Specifically, element $i \times \mathbf{ia1}$ of **a** must contain the coefficient a_i, for $i = 0, 1, \ldots, n$. Only these $n + 1$ elements will be accessed.

5: **ia1** – Integer *Input*

On entry: the index increment of **a**. Most frequently the Chebyshev coefficients are stored in adjacent elements of **a**, and **ia1** must be set to 1. However, if for example, they are stored in $\mathbf{a}[0], \mathbf{a}[3], \mathbf{a}[6], \ldots$, then the value of **ia1** must be 3. See also Section 9.

Constraint: **ia1** ≥ 1.

6: **qatm1** – double *Input*

On entry: the value that the integrated polynomial is required to have at the lower end point of its interval of definition, i.e., at $\bar{x} = -1$ which corresponds to $x = x_{\text{min}}$. Thus, **qatm1** is a constant of integration and will normally be set to zero by you.

7: **aintc**[*dim*] – double *Output*

Note: the dimension, *dim*, of the array **aintc** must be at least $(1 + (\mathbf{n} + 1) \times \mathbf{iaint1})$.

On exit: the Chebyshev coefficients of the integral $q(x)$. (The integration is with respect to the variable x, and the constant coefficient is chosen so that $q(x_{\text{min}})$ equals **qatm1**). Specifically, element $i \times \mathbf{iaint1}$ of **aintc** contains the coefficient a'_i, for $i = 0, 1, \ldots, n + 1$.

8: **iaint1** – Integer *Input*

On entry: the index increment of **aintc**. Most frequently the Chebyshev coefficients are required in adjacent elements of **aintc**, and **iaint1** must be set to 1. However, if, for example, they are to be stored in $\mathbf{aintc}[0], \mathbf{aintc}[3], \mathbf{aintc}[6], \ldots$, then the value of **iaint1** must be 3. See also Section 9.

Constraint: **iaint1** ≥ 1.

9: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_BAD_PARAM

On entry, argument ⟨*value*⟩ had an illegal value.

NE_INT

On entry, **ia1** = ⟨*value*⟩.
Constraint: **ia1** ≥ 1.

On entry, **iaint1** = ⟨*value*⟩.
Constraint: **iaint1** ≥ 1.

On entry, $\mathbf{n} + 1$ = ⟨*value*⟩.
Constraint: $\mathbf{n} + 1 \geq 1$.

On entry, **n** = ⟨*value*⟩.
Constraint: $\mathbf{n} \geq 0$.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

NE_REAL_2

On entry, **xmax** = ⟨*value*⟩ and **xmin** = ⟨*value*⟩.
Constraint: **xmax** > **xmin**.

7 Accuracy

In general there is a gain in precision in numerical integration, in this case associated with the division by $2i$ in the formula quoted in Section 3.

8 Parallelism and Performance

Not applicable.

9 Further Comments

The time taken is approximately proportional to $n + 1$.

The increments **ia1**, **iaint1** are included as arguments to give a degree of flexibility which, for example, allows a polynomial in two variables to be integrated with respect to either variable without rearranging the coefficients.

10 Example

Suppose a polynomial has been computed in Chebyshev series form to fit data over the interval $[-0.5, 2.5]$. The following program evaluates the integral of the polynomial from 0.0 to 2.0. (For the purpose of this example, **xmin**, **xmax** and the Chebyshev coefficients are simply supplied . Normally a program would read in or generate data and compute the fitted polynomial).

10.1 Program Text

```
/* nag_1d_cheb_intg (e02ajc) Example Program.
 *
 * Copyright 2001 Numerical Algorithms Group.
 *
 * Mark 7, 2001.
 */
```

```
#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nage02.h>

int main(void)
{
  /* Initialized data */
  const double xmin = -0.5;
  const double xmax = 2.5;
  const double a[7] =
  { 2.53213, 1.13032, 0.2715, 0.04434, 0.00547, 5.4e-4, 4e-5 };

  /* Scalars */
  double       ra, rb, result, xa, xb, zero;
  Integer      exit_status, n, one;
  NagError     fail;

  /* Arrays */
  double       *aint = 0;

  INIT_FAIL(fail);

  exit_status = 0;
  printf("nag_1d_cheb_intg (e02ajc) Example Program Results\n");

  n = 6;
  zero = 0.0;
  one = 1;

  /* Allocate memory */
  if (!(aint = NAG_ALLOC(n + 2, double)))
    {
      printf("Allocation failure\n");
      exit_status = -1;
      goto END;
    }

  /* nag_1d_cheb_intg (e02ajc).
   * Integral of fitted polynomial in Chebyshev series form
   */
  nag_1d_cheb_intg(n, xmin, xmax, a, one, zero, aint, one, &fail);
  if (fail.code != NE_NOERROR)
    {
      printf("Error from nag_1d_cheb_intg (e02ajc).\n%s\n",
              fail.message);
      exit_status = 1;
      goto END;
    }

  xa = 0.0;
  xb = 2.0;
  /* nag_1d_cheb_eval2 (e02akc).
   * Evaluation of fitted polynomial in one variable from
   * Chebyshev series form
   */
  nag_1d_cheb_eval2(n+1, xmin, xmax, aint, one, xa, &ra, &fail);
  if (fail.code != NE_NOERROR)
    {
      printf("Error from nag_1d_cheb_eval2 (e02akc).\n%s\n",
              fail.message);
      exit_status = 1;
      goto END;
    }

  /* nag_1d_cheb_eval2 (e02akc), see above. */
  nag_1d_cheb_eval2(n+1, xmin, xmax, aint, one, xb, &rb, &fail);
  if (fail.code != NE_NOERROR)
    {
      printf("Error from nag_1d_cheb_eval2 (e02akc).\n%s\n",
              fail.message);
```

```
      exit_status = 1;
      goto END;
    }

  result = rb - ra;
  printf("\n");
  printf("Value of definite integral is %10.4f\n", result);
 END:
  NAG_FREE(aint);

  return exit_status;
}
```

10.2 Program Data

None.

10.3 Program Results

```
nag_1d_cheb_intg (e02ajc) Example Program Results

Value of definite integral is     2.1515
```