NAG Toolbox

nag_specfun_opt_heston_greeks (s30nb)

1 Purpose

nag_specfun_opt_heston_greeks (s30nb) computes the European option price given by Heston's stochastic volatility model together with its sensitivities (Greeks).

2 Syntax

```
[p, delta, gamma, vega, theta, rho, vanna, charm, speed, zomma, vomma, ifail] =
nag_specfun_opt_heston_greeks(calput, x, s, t, sigmav, kappa, corr, var0, eta,
grisk, r, q, 'm', m, 'n', n)
```

```
[p, delta, gamma, vega, theta, rho, vanna, charm, speed, zomma, vomma, ifail] =
s30nb(calput, x, s, t, sigmav, kappa, corr, var0, eta, grisk, r, q, 'm', m, 'n',
n)
```

3 Description

nag_specfun_opt_heston_greeks (s30nb) computes the price and sensitivities of a European option using Heston's stochastic volatility model. The return on the asset price, S, is

$$\frac{dS}{S} = (r-q)dt + \sqrt{v_t}dW_t^{(1)}$$

and the instantaneous variance, v_t, is defined by a mean-reverting square root stochastic process,

$$dv_t = \kappa(\eta - v_t)dt + \sigma_v\sqrt{v_t}dW_t^{(2)},$$

where r is the risk free annual interest rate; q is the annual dividend rate; v_t is the variance of the asset price; σ_v is the volatility of the volatility, $\sqrt{v_t}$; κ is the mean reversion rate; η is the long term variance. $dW_t^{(i)}$, for $i = 1, 2$, denotes two correlated standard Brownian motions with

$$\mathbb{C}\mathrm{ov}\left[dW_t^{(1)}, dW_t^{(2)}\right] = \rho dt.$$

The option price is computed by evaluating the integral transform given by Lewis (2000) using the form of the characteristic function discussed by Albrecher *et al.* (2007), see also Kilin (2006).

$$P_{\text{call}} = Se^{-qT} - Xe^{-rT}\frac{1}{\pi}\text{Re}\left[\int_{0+i/2}^{\infty+i/2} e^{-ik\bar{X}}\frac{\hat{H}(k,v,T)}{k^2-ik}\text{d}k\right], \tag{1}$$

where $\bar{X} = \ln(S/X) + (r-q)T$ and

$$\hat{H}(k,v,T) = \exp\left(\frac{2\kappa\eta}{\sigma_v^2}\left[t\text{gendgroup} - \ln\left(\frac{1-\text{he}^{-\xi\text{t}}}{1-\text{h}}\right)\right] + \text{v}_\text{t}\text{g}\left[\frac{1-\text{e}^{-\xi\text{t}}}{1-\text{he}^{-\xi\text{t}}}\right]\right),$$

$$g = \frac{1}{2}(b-\xi), \quad h = \frac{b-\xi}{b+\xi}, \quad t = \sigma_v^2 T/2,$$

$$\xi = \left[b^2 + 4\frac{k^2-ik}{\sigma_v^2}\right]^{\frac{1}{2}},$$

$$b = \frac{2}{\sigma_v^2}\left[(1-\gamma+ik)\rho\sigma_v + \sqrt{\kappa^2 - \gamma(1-\gamma)\sigma_v^2}\right]$$

with $t = \sigma_v^2 T/2$. Here γ is the risk aversion parameter of the representative agent with $0 \leq \gamma \leq 1$ and

$\gamma(1-\gamma)\sigma_v^2 \leq \kappa^2$. The value $\gamma = 1$ corresponds to $\lambda = 0$, where λ is the market price of risk in Heston (1993) (see Lewis (2000) and Rouah and Vainberg (2007)).

The price of a put option is obtained by put-call parity.

$$P_{\text{put}} = P_{\text{call}} + Xe^{-rT} - Se^{-qT}.$$

Writing the expression for the price of a call option as

$$P_{\text{call}} = Se^{-qT} - Xe^{-rT}\frac{1}{\pi}\text{Re}\left[\int_{0+i/2}^{\infty+i/2} I(k,r,S,T,v)\mathrm{d}k\right]$$

then the sensitivities or Greeks can be obtained in the following manner,

Delta

$$\frac{\partial P_{\text{call}}}{\partial S} = e^{-qT} + \frac{Xe^{-rT}}{S}\frac{1}{\pi}\text{Re}\left[\int_{0+i/2}^{\infty+i/2}(ik)I(k,r,S,T,v)\mathrm{d}k\right],$$

Vega

$$\frac{\partial P}{\partial v} = -Xe^{-rT}\frac{1}{\pi}\text{Re}\left[\int_{0-i/2}^{0+i/2} f_2 I(k,r,j,S,T,v)\mathrm{d}k\right], \quad \text{where } f_2 = g\left[\frac{1-e^{-\xi t}}{1-he^{-\xi t}}\right],$$

Rho

$$\frac{\partial P_{\text{call}}}{\partial r} = TXe^{-rT}\frac{1}{\pi}\text{Re}\left[\int_{0+i/2}^{\infty+i/2}(1+ik)I(k,r,S,T,v)\mathrm{d}k\right].$$

The option price $P_{ij} = P(X = X_i, T = T_j)$ is computed for each strike price in a set X_i, $i = 1,2,\ldots,m$, and for each expiry time in a set T_j, $j = 1,2,\ldots,n$.

4 References

Albrecher H, Mayer P, Schoutens W and Tistaert J (2007) The little Heston trap *Wilmott Magazine* **January 2007** 83–92

Heston S (1993) A closed-form solution for options with stochastic volatility with applications to bond and currency options *Review of Financial Studies* **6** 327–343

Kilin F (2006) Accelerating the calibration of stochastic volatility models *MPRA Paper No. 2975* http://mpra.ub.uni-muenchen.de/2975/

Lewis A L (2000) Option valuation under stochastic volatility *Finance Press, USA*

Rouah F D and Vainberg G (2007) *Option Pricing Models and Volatility using Excel-VBA* John Wiley and Sons, Inc

5 Parameters

5.1 Compulsory Input Parameters

1: **calput** – CHARACTER(1)

Determines whether the option is a call or a put.

calput = 'C'
: A call; the holder has a right to buy.

calput = 'P'
: A put; the holder has a right to sell.

Constraint: **calput** = 'C' or 'P'.

2: **x**(**m**) – REAL (KIND=nag_wp) array

x(i) must contain X_i, the ith strike price, for $i = 1, 2, \ldots, \mathbf{m}$.

Constraint: **x**(i) $\geq z$ and **x**(i) $\leq 1/z$, where $z = $ x02am(), the safe range parameter, for $i = 1, 2, \ldots, \mathbf{m}$.

3: **s** – REAL (KIND=nag_wp)

S, the price of the underlying asset.

Constraint: **s** $\geq z$ and **s** $\leq 1.0/z$, where $z = $ x02am(), the safe range parameter.

4: **t**(**n**) – REAL (KIND=nag_wp) array

t(i) must contain T_i, the ith time, in years, to expiry, for $i = 1, 2, \ldots, \mathbf{n}$.

Constraint: **t**(i) $\geq z$, where $z = $ x02am(), the safe range parameter, for $i = 1, 2, \ldots, \mathbf{n}$.

5: **sigmav** – REAL (KIND=nag_wp)

The volatility, σ_v, of the volatility process, $\sqrt{v_t}$. Note that a rate of 20% should be entered as 0.2.

Constraint: **sigmav** > 0.0.

6: **kappa** – REAL (KIND=nag_wp)

κ, the long term mean reversion rate of the volatility.

Constraint: **kappa** > 0.0.

7: **corr** – REAL (KIND=nag_wp)

The correlation between the two standard Brownian motions for the asset price and the volatility.

Constraint: $-1.0 \leq$ **corr** ≤ 1.0.

8: **var0** – REAL (KIND=nag_wp)

The initial value of the variance, v_t, of the asset price.

Constraint: **var0** ≥ 0.0.

9: **eta** – REAL (KIND=nag_wp)

η, the long term mean of the variance of the asset price.

Constraint: **eta** > 0.0.

10: **grisk** – REAL (KIND=nag_wp)

The risk aversion parameter, γ, of the representative agent.

Constraint: $0.0 \leq$ **grisk** ≤ 1.0 and **grisk** $\times$ $(1 -$ **grisk**$)$ $\times$ **sigmav** $\times$ **sigmav** $\leq$ **kappa** $\times$ **kappa**.

11: **r** – REAL (KIND=nag_wp)

r, the annual risk-free interest rate, continuously compounded. Note that a rate of 5% should be entered as 0.05.

Constraint: **r** ≥ 0.0.

12: **q** – REAL (KIND=nag_wp)

q, the annual continuous yield rate. Note that a rate of 8% should be entered as 0.08.

Constraint: **q** ≥ 0.0.

5.2 Optional Input Parameters

1: **m** – INTEGER

Default: the dimension of the array **x**.

The number of strike prices to be used.

Constraint: $\mathbf{m} \geq 1$.

2: **n** – INTEGER

Default: the dimension of the array **t**.

The number of times to expiry to be used.

Constraint: $\mathbf{n} \geq 1$.

5.3 Output Parameters

1: $\mathbf{p}(ldp, \mathbf{n})$ – REAL (KIND=nag_wp) array

$ldp = \mathbf{m}$.

$\mathbf{p}(i, j)$ contains P_{ij}, the option price evaluated for the strike price $\mathbf{x}_i$ at expiry $\mathbf{t}_j$ for $i = 1, 2, \ldots, \mathbf{m}$ and $j = 1, 2, \ldots, \mathbf{n}$.

2: $\mathbf{delta}(ldp, \mathbf{n})$ – REAL (KIND=nag_wp) array

$ldp = \mathbf{m}$.

The leading $\mathbf{m} \times \mathbf{n}$ part of the array **delta** contains the sensitivity, $\frac{\partial P}{\partial S}$, of the option price to change in the price of the underlying asset.

3: $\mathbf{gamma}(ldp, \mathbf{n})$ – REAL (KIND=nag_wp) array

$ldp = \mathbf{m}$.

The leading $\mathbf{m} \times \mathbf{n}$ part of the array **gamma** contains the sensitivity, $\frac{\partial^2 P}{\partial S^2}$, of **delta** to change in the price of the underlying asset.

4: $\mathbf{vega}(ldp, \mathbf{n})$ – REAL (KIND=nag_wp) array

$ldp = \mathbf{m}$.

$\mathbf{vega}(i, j)$, contains the first-order Greek measuring the sensitivity of the option price P_{ij} to change in the volatility of the underlying asset, i.e., $\frac{\partial P_{ij}}{\partial \sigma}$, for $i = 1, 2, \ldots, \mathbf{m}$ and $j = 1, 2, \ldots, \mathbf{n}$.

5: $\mathbf{theta}(ldp, \mathbf{n})$ – REAL (KIND=nag_wp) array

$ldp = \mathbf{m}$.

$\mathbf{theta}(i, j)$, contains the first-order Greek measuring the sensitivity of the option price P_{ij} to change in time, i.e., $-\frac{\partial P_{ij}}{\partial T}$, for $i = 1, 2, \ldots, \mathbf{m}$ and $j = 1, 2, \ldots, \mathbf{n}$, where $b = r - q$.

6: $\mathbf{rho}(ldp, \mathbf{n})$ – REAL (KIND=nag_wp) array

$ldp = \mathbf{m}$.

$\mathbf{rho}(i, j)$, contains the first-order Greek measuring the sensitivity of the option price P_{ij} to change in the annual risk-free interest rate, i.e., $-\frac{\partial P_{ij}}{\partial r}$, for $i = 1, 2, \ldots, \mathbf{m}$ and $j = 1, 2, \ldots, \mathbf{n}$.

7: $\mathbf{vanna}(ldp, \mathbf{n})$ – REAL (KIND=nag_wp) array

$ldp = \mathbf{m}$.

vanna(i, j), contains the second-order Greek measuring the sensitivity of the first-order Greek Δ_{ij} to change in the volatility of the asset price, i.e., $-\frac{\partial \Delta_{ij}}{\partial T} = -\frac{\partial^2 P_{ij}}{\partial S \partial \sigma}$, for $i = 1, 2, \ldots,$ **m** and $j = 1, 2, \ldots,$ **n**.

8: **charm**$(ldp,$ **n**$)$ – REAL (KIND=nag_wp) array

$ldp =$ **m**.

charm(i, j), contains the second-order Greek measuring the sensitivity of the first-order Greek Δ_{ij} to change in the time, i.e., $-\frac{\partial \Delta_{ij}}{\partial T} = -\frac{\partial^2 P_{ij}}{\partial S \partial T}$, for $i = 1, 2, \ldots,$ **m** and $j = 1, 2, \ldots,$ **n**.

9: **speed**$(ldp,$ **n**$)$ – REAL (KIND=nag_wp) array

$ldp =$ **m**.

speed(i, j), contains the third-order Greek measuring the sensitivity of the second-order Greek Γ_{ij} to change in the price of the underlying asset, i.e., $-\frac{\partial \Gamma_{ij}}{\partial S} = -\frac{\partial^3 P_{ij}}{\partial S^3}$, for $i = 1, 2, \ldots,$ **m** and $j = 1, 2, \ldots,$ **n**.

10: **zomma**$(ldp,$ **n**$)$ – REAL (KIND=nag_wp) array

$ldp =$ **m**.

zomma(i, j), contains the third-order Greek measuring the sensitivity of the second-order Greek Γ_{ij} to change in the volatility of the underlying asset, i.e., $-\frac{\partial \Gamma_{ij}}{\partial \sigma} = -\frac{\partial^3 P_{ij}}{\partial S^2 \partial \sigma}$, for $i = 1, 2, \ldots,$ **m** and $j = 1, 2, \ldots,$ **n**.

11: **vomma**$(ldp,$ **n**$)$ – REAL (KIND=nag_wp) array

$ldp =$ **m**.

vomma(i, j), contains the second-order Greek measuring the sensitivity of the first-order Greek Δ_{ij} to change in the volatility of the underlying asset, i.e., $-\frac{\partial \Delta_{ij}}{\partial \sigma} = -\frac{\partial^2 P_{ij}}{\partial \sigma^2}$, for $i = 1, 2, \ldots,$ **m** and $j = 1, 2, \ldots,$ **n**.

12: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, **calput** $= \langle value \rangle$ was an illegal value.

ifail $= 2$

Constraint: **m** ≥ 1.

ifail $= 3$

Constraint: **n** ≥ 1.

ifail $= 4$

Constraint: **x**$(i) \geq \langle value \rangle$ and **x**$(i) \leq \langle value \rangle$.

ifail $= 5$

Constraint: **s** $\geq \langle value \rangle$ and **s** $\leq \langle value \rangle$.

ifail $= 6$

Constraint: $\mathbf{t}(i) \geq \langle value \rangle$.

ifail $= 7$

Constraint: **sigmav** > 0.0.

ifail $= 8$

Constraint: **kappa** > 0.0.

ifail $= 9$

Constraint: $|\mathbf{corr}| \leq 1.0$.

ifail $= 10$

Constraint: **var0** ≥ 0.0.

ifail $= 11$

Constraint: **eta** > 0.0.

ifail $= 12$

Constraint: $0.0 \leq \mathbf{grisk} \leq 1.0$ and $\mathbf{grisk} \times (1.0 - \mathbf{grisk}) \times \mathbf{sigmav}^2 \leq \mathbf{kappa}^2$.

ifail $= 13$

Constraint: **r** ≥ 0.0.

ifail $= 14$

Constraint: **q** ≥ 0.0.

ifail $= 16$

Constraint: $ldp \geq \mathbf{m}$.

ifail $= 17$ (*warning*)

Quadrature has not converged to the required accuracy. However, the result should be a reasonable approximation.

ifail $= 18$ (*warning*)

Solution cannot be computed accurately. Check values of input arguments.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

The accuracy of the output is determined by the accuracy of the numerical quadrature used to evaluate the integral in (1). An adaptive method is used which evaluates the integral to within a tolerance of $\max\left(10^{-8}, 10^{-10} \times |I|\right)$, where $|I|$ is the absolute value of the integral.

8 Further Comments

None.

9 Example

This example computes the price and sensitivities of a European call using Heston's stochastic volatility model. The time to expiry is 1 year, the stock price is 100 and the strike price is 100. The risk-free interest rate is 2.5% per year, the volatility of the variance, σ_v, is 57.51% per year, the mean reversion parameter, κ, is 1.5768, the long term mean of the variance, η, is 0.0398 and the correlation between the volatility process and the stock price process, ρ, is -0.5711. The risk aversion parameter, γ, is 1.0 and the initial value of the variance, **var0**, is 0.0175.

9.1 Program Text

```
     function s30nb_example

fprintf('s30nb example results\n\n');

calput = 'C';
s      = 100.0;
r      = 0.025;
q      = 0.0;
kappa  = 1.5768;
eta    = 0.0398;
var0   = 0.0175;
sigmav = 0.5751;
corr   = -0.5711;
grisk  = 1;
x      = [100.0];
t      = [1];

[p, delta, gamma, vega,  theta, rho, ...
    vanna, charm, speed, zomma, vomma, ifail] = ...
    s30nb(...
          calput, x, s, t, sigmav, kappa, corr, var0, eta, grisk, r, q);

fprintf('\nHeston''s Stochastic Volatility Model\n');
if calput == 'C' || calput == 'c'
  fprintf('European Call :\n');
else
  fprintf('European Put :\n');
end
fprintf(' Spot                    =  %9.4f\n', s);
fprintf(' Volatility of vol       =  %9.4f\n', sigmav);
fprintf(' Mean reversion          =  %9.4f\n', kappa);
fprintf(' Correlation             =  %9.4f\n', corr);
fprintf(' Variance                =  %9.4f\n', var0);
fprintf(' Mean of variance        =  %9.4f\n', eta);
fprintf(' Risk aversion           =  %9.4f\n', grisk);
fprintf(' Rate                    =  %9.4f\n', r);
fprintf(' Dividend                =  %9.4f\n\n', q);

for j=1:1
  fprintf('%8s%9s%9s%9s%9s%9s%9s\n','Strike','Price','Delta','Gamma',...
          'Vega','Theta','Rho');
  for i=1:1
    fprintf('%8.4f%9.4f%9.4f%9.4f%9.4f%9.4f%9.2f\n', x(i), p(i,j), ...
          delta(i,j), gamma(i,j), vega(i,j), theta(i,j), rho(i,j));
  end
  fprintf('\n%26s%9s%9s%9s%9s\n','Vanna','Charm','Speed','Zomma','Vomma');
  for i=1:1
    fprintf('%17s%9.4f%9.4f%9.4f%9.4f%9.2f\n', ' ', vanna(i,j), ...
            charm(i,j), speed(i,j), zomma(i,j), vomma(i,j));
  end
end
```

9.2 Program Results

```
     s30nb example results

Heston's Stochastic Volatility Model
European Call :
 Spot                   =   100.0000
 Volatility of vol      =     0.5751
 Mean reversion         =     1.5768
 Correlation            =    -0.5711
 Variance               =     0.0175
 Mean of variance       =     0.0398
 Risk aversion          =     1.0000
 Rate                   =     0.0250
 Dividend               =     0.0000

  Strike    Price    Delta    Gamma     Vega    Theta      Rho
100.0000   7.2743   0.6945   0.0251  52.5461  -4.9969    62.17

                     Vanna    Charm    Speed    Zomma    Vomma
                   -0.5643  -0.0321  -0.0023  -0.1976  -321.08
```