

## NAG Toolbox

### nag\_specfun\_opt\_heston\_price (s30na)

#### 1 Purpose

nag\_specfun\_opt\_heston\_price (s30na) computes the European option price given by Heston's stochastic volatility model.

#### 2 Syntax

```
[p, ifail] = nag_specfun_opt_heston_price(calput, x, s, t, sigmav, kappa, corr,
var0, eta, grisk, r, q, 'm', m, 'n', n)
```

```
[p, ifail] = s30na(calput, x, s, t, sigmav, kappa, corr, var0, eta, grisk, r, q,
'm', m, 'n', n)
```

#### 3 Description

nag\_specfun\_opt\_heston\_price (s30na) computes the price of a European option using Heston's stochastic volatility model. The return on the asset price,  $S$ , is

$$\frac{dS}{S} = (r - q)dt + \sqrt{v_t}dW_t^{(1)}$$

and the instantaneous variance,  $v_t$ , is defined by a mean-reverting square root stochastic process,

$$dv_t = \kappa(\eta - v_t)dt + \sigma_v\sqrt{v_t}dW_t^{(2)},$$

where  $r$  is the risk free annual interest rate;  $q$  is the annual dividend rate;  $v_t$  is the variance of the asset price;  $\sigma_v$  is the volatility of the volatility,  $\sqrt{v_t}$ ;  $\kappa$  is the mean reversion rate;  $\eta$  is the long term variance.  $dW_t^{(i)}$ , for  $i = 1, 2$ , denotes two correlated standard Brownian motions with

$$\text{Cov}\left[dW_t^{(1)}, dW_t^{(2)}\right] = \rho dt.$$

The option price is computed by evaluating the integral transform given by Lewis (2000) using the form of the characteristic function discussed by Albrecher *et al.* (2007), see also Kilin (2006).

$$P_{\text{call}} = Se^{-qT} - Xe^{-rT} \frac{1}{\pi} \text{Re} \left[ \int_{0+i/2}^{\infty+i/2} e^{-ik\bar{X}} \frac{\hat{H}(k, v, T)}{k^2 - ik} dk \right], \quad (1)$$

where  $\bar{X} = \ln(S/X) + (r - q)T$  and

$$\hat{H}(k, v, T) = \exp\left(\frac{2\kappa\eta}{\sigma_v^2} \left[ \text{tgendgroup} - \ln\left(\frac{1 - h e^{-\xi t}}{1 - h}\right) \right] + v_t g \left[ \frac{1 - e^{-\xi t}}{1 - h e^{-\xi t}} \right] \right),$$

$$g = \frac{1}{2}(b - \xi), \quad h = \frac{b - \xi}{b + \xi}, \quad t = \sigma_v^2 T / 2,$$

$$\xi = \left[ b^2 + 4 \frac{k^2 - ik}{\sigma_v^2} \right]^{\frac{1}{2}},$$

$$b = \frac{2}{\sigma_v^2} \left[ (1 - \gamma + ik)\rho\sigma_v + \sqrt{\kappa^2 - \gamma(1 - \gamma)\sigma_v^2} \right]$$

with  $t = \sigma_v^2 T / 2$ . Here  $\gamma$  is the risk aversion parameter of the representative agent with  $0 \leq \gamma \leq 1$  and  $\gamma(1 - \gamma)\sigma_v^2 \leq \kappa^2$ . The value  $\gamma = 1$  corresponds to  $\lambda = 0$ , where  $\lambda$  is the market price of risk in Heston (1993) (see Lewis (2000) and Rouah and Vainberg (2007)).

The price of a put option is obtained by put-call parity.

The option price  $P_{ij} = P(X = X_i, T = T_j)$  is computed for each strike price in a set  $X_i$ ,  $i = 1, 2, \dots, m$ , and for each expiry time in a set  $T_j$ ,  $j = 1, 2, \dots, n$ .

## 4 References

Albrecher H, Mayer P, Schoutens W and Tistaert J (2007) The little Heston trap *Wilmott Magazine* **January 2007** 83–92

Heston S (1993) A closed-form solution for options with stochastic volatility with applications to bond and currency options *Review of Financial Studies* **6** 327–343

Kilin F (2006) Accelerating the calibration of stochastic volatility models *MPRA Paper No. 2975* <http://mpra.ub.uni-muenchen.de/2975/>

Lewis A L (2000) Option valuation under stochastic volatility *Finance Press, USA*

Rouah F D and Vainberg G (2007) *Option Pricing Models and Volatility using Excel-VBA* John Wiley and Sons, Inc

## 5 Parameters

### 5.1 Compulsory Input Parameters

1: **calput** – CHARACTER(1)

Determines whether the option is a call or a put.

**calput** = 'C'

A call; the holder has a right to buy.

**calput** = 'P'

A put; the holder has a right to sell.

*Constraint:* **calput** = 'C' or 'P'.

2: **x(m)** – REAL (KIND=nag\_wp) array

**x(i)** must contain  $X_i$ , the  $i$ th strike price, for  $i = 1, 2, \dots, m$ .

*Constraint:* **x(i)**  $\geq z$  and **x(i)**  $\leq 1/z$ , where  $z = x02am()$ , the safe range parameter, for  $i = 1, 2, \dots, m$ .

3: **s** – REAL (KIND=nag\_wp)

$S$ , the price of the underlying asset.

*Constraint:* **s**  $\geq z$  and **s**  $\leq 1.0/z$ , where  $z = x02am()$ , the safe range parameter.

4: **t(n)** – REAL (KIND=nag\_wp) array

**t(i)** must contain  $T_i$ , the  $i$ th time, in years, to expiry, for  $i = 1, 2, \dots, n$ .

*Constraint:* **t(i)**  $\geq z$ , where  $z = x02am()$ , the safe range parameter, for  $i = 1, 2, \dots, n$ .

5: **sigmav** – REAL (KIND=nag\_wp)

The volatility,  $\sigma_v$ , of the volatility process,  $\sqrt{v_t}$ . Note that a rate of 20% should be entered as 0.2.

*Constraint:* **sigmav**  $> 0.0$ .

6: **kappa** – REAL (KIND=nag\_wp)

$\kappa$ , the long term mean reversion rate of the volatility.

*Constraint:* **kappa**  $> 0.0$ .

- 7: **corr** – REAL (KIND=nag\_wp)  
The correlation between the two standard Brownian motions for the asset price and the volatility.  
*Constraint:*  $-1.0 \leq \mathbf{corr} \leq 1.0$ .
- 8: **var0** – REAL (KIND=nag\_wp)  
The initial value of the variance,  $v_t$ , of the asset price.  
*Constraint:*  $\mathbf{var0} \geq 0.0$ .
- 9: **eta** – REAL (KIND=nag\_wp)  
 $\eta$ , the long term mean of the variance of the asset price.  
*Constraint:*  $\mathbf{eta} > 0.0$ .
- 10: **grisk** – REAL (KIND=nag\_wp)  
The risk aversion parameter,  $\gamma$ , of the representative agent.  
*Constraint:*  $0.0 \leq \mathbf{grisk} \leq 1.0$  and  $\mathbf{grisk} \times (1.0 - \mathbf{grisk}) \times \mathbf{sigmav} \times \mathbf{sigmav} \leq \mathbf{kappa} \times \mathbf{kappa}$ .
- 11: **r** – REAL (KIND=nag\_wp)  
 $r$ , the annual risk-free interest rate, continuously compounded. Note that a rate of 5% should be entered as 0.05.  
*Constraint:*  $\mathbf{r} \geq 0.0$ .
- 12: **q** – REAL (KIND=nag\_wp)  
 $q$ , the annual continuous yield rate. Note that a rate of 8% should be entered as 0.08.  
*Constraint:*  $\mathbf{q} \geq 0.0$ .

## 5.2 Optional Input Parameters

- 1: **m** – INTEGER  
*Default:* the dimension of the array **x**.  
The number of strike prices to be used.  
*Constraint:*  $\mathbf{m} \geq 1$ .
- 2: **n** – INTEGER  
*Default:* the dimension of the array **t**.  
The number of times to expiry to be used.  
*Constraint:*  $\mathbf{n} \geq 1$ .

## 5.3 Output Parameters

- 1: **p**(*ldp*, **n**) – REAL (KIND=nag\_wp) array  
*ldp* = **m**.  
**p**(*i*, *j*) contains  $P_{ij}$ , the option price evaluated for the strike price  $\mathbf{x}_i$  at expiry  $\mathbf{t}_j$  for  $i = 1, 2, \dots, \mathbf{m}$  and  $j = 1, 2, \dots, \mathbf{n}$ .
- 2: **ifail** – INTEGER  
**ifail** = 0 unless the function detects an error (see Section 5).

## 6 Error Indicators and Warnings

Errors or warnings detected by the function:

**ifail** = 1

On entry, **calput** =  $\langle value \rangle$  was an illegal value.

**ifail** = 2

Constraint: **m**  $\geq$  1.

**ifail** = 3

Constraint: **n**  $\geq$  1.

**ifail** = 4

Constraint: **x**(*i*)  $\geq$   $\langle value \rangle$  and **x**(*i*)  $\leq$   $\langle value \rangle$ .

**ifail** = 5

Constraint: **s**  $\geq$   $\langle value \rangle$  and **s**  $\leq$   $\langle value \rangle$ .

**ifail** = 6

Constraint: **t**(*i*)  $\geq$   $\langle value \rangle$ .

**ifail** = 7

Constraint: **sigmav**  $>$  0.0.

**ifail** = 8

Constraint: **kappa**  $>$  0.0.

**ifail** = 9

Constraint: **|corr**|  $\leq$  1.0.

**ifail** = 10

Constraint: **var0**  $\geq$  0.0.

**ifail** = 11

Constraint: **eta**  $>$  0.0.

**ifail** = 12

Constraint:  $0.0 \leq \mathbf{grisk} \leq 1.0$  and  $\mathbf{grisk} \times (1.0 - \mathbf{grisk}) \times \mathbf{sigmav}^2 \leq \mathbf{kappa}^2$ .

**ifail** = 13

Constraint: **r**  $\geq$  0.0.

**ifail** = 14

Constraint: **q**  $\geq$  0.0.

**ifail** = 16

Constraint: *ldp*  $\geq$  **m**.

**ifail** = 17 (*warning*)

Quadrature has not converged to the specified accuracy. However, the result should be a reasonable approximation.

**ifail** = 18 (*warning*)

Solution cannot be computed accurately. Check values of input arguments.

**ifail** = -99

An unexpected error has been triggered by this routine. Please contact NAG.

**ifail** = -399

Your licence key may have expired or may not have been installed correctly.

**ifail** = -999

Dynamic memory allocation failed.

## 7 Accuracy

The accuracy of the output is determined by the accuracy of the numerical quadrature used to evaluate the integral in (1). An adaptive method is used which evaluates the integral to within a tolerance of  $\max(10^{-8}, 10^{-10} \times |I|)$ , where  $|I|$  is the absolute value of the integral.

## 8 Further Comments

None.

## 9 Example

This example computes the price of a European call using Heston's stochastic volatility model. The time to expiry is 6 months, the stock price is 100 and the strike price is 100. The risk-free interest rate is 5% per year, the volatility of the variance,  $\sigma_v$ , is 22.5% per year, the mean reversion parameter,  $\kappa$ , is 2.0, the long term mean of the variance,  $\eta$ , is 0.01 and the correlation between the volatility process and the stock price process,  $\rho$ , is 0.0. The risk aversion parameter,  $\gamma$ , is 1.0 and the initial value of the variance, **var0**, is 0.01.

### 9.1 Program Text

```
function s30na_example

fprintf('s30na example results\n\n');

put = 'C';
s = 100.0;
r = 0.05;
q = 0.0;
gamma=1.0;
kappa = 2.0;
eta = 0.01;
var0 = 0.01;
sigmav = 0.225;
corr = 0.0;
x = [100.0];
t = [0.5];

[p, ifail] = s30na( ...
    put, x, s, t, sigmav, kappa, corr, var0, eta, gamma, r, q);

fprintf('\nHeston''s Stochastic Volatility Model\n European Call :\n');
fprintf(' Spot           = %9.4f\n', s);
```

```

fprintf(' Volatility of vol      = %9.4f\n', sigmav);
fprintf(' Mean reversion        = %9.4f\n', kappa);
fprintf(' Correlation              = %9.4f\n', corr);
fprintf(' Variance                  = %9.4f\n', eta);
fprintf(' Mean of variance          = %9.4f\n', var0);
fprintf(' Risk aversion             = %9.4f\n', gamma);
fprintf(' Rate                      = %9.4f\n', r);
fprintf(' Dividend                  = %9.4f\n\n', 1);

fprintf('   Strike   Expiry   Option Price\n');
for i=1:1
    for j=1:1
        fprintf('%9.4f %9.4f %9.4f\n', x(i), t(j), p(i,j));
    end
end
end

```

## 9.2 Program Results

s30na example results

```

Heston's Stochastic Volatility Model
European Call :
Spot                = 100.0000
Volatility of vol   = 0.2250
Mean reversion      = 2.0000
Correlation         = 0.0000
Variance            = 0.0100
Mean of variance    = 0.0100
Risk aversion       = 1.0000
Rate                = 0.0500
Dividend            = 1.0000

   Strike   Expiry   Option Price
100.0000   0.5000   4.0851

```

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