NAG Toolbox

nag_specfun_opt_binary_con_price (s30ca)

1 Purpose

nag_specfun_opt_binary_con_price (s30ca) computes the price of a binary or digital cash-or-nothing option.

2 Syntax

```
[p, ifail] = nag_specfun_opt_binary_con_price(calput, x, s, k, t, sigma, r, q, 'm', m, 'n', n)
[p, ifail] = s30ca(calput, x, s, k, t, sigma, r, q, 'm', m, 'n', n)
```

3 Description

nag_specfun_opt_binary_con_price (s30ca) computes the price of a binary or digital cash-or-nothing option which pays a fixed amount, K, at expiration if the option is in-the-money (see Section 2.4 in the S Chapter Introduction). For a strike price, X, underlying asset price, S, and time to expiry, T, the payoff is therefore K, if $S > X$ for a call or $S < X$ for a put. Nothing is paid out when this condition is not met.

The price of a call with volatility, σ, risk-free interest rate, r, and annualised dividend yield, q, is

$$P_{\text{call}} = Ke^{-rT}\Phi(d_2)$$

and for a put,

$$P_{\text{put}} = Ke^{-rT}\Phi(-d_2)$$

where Φ is the cumulative Normal distribution function,

$$\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x}\left(-y^2/2\right)dy,$$

and

$$d_2 = \frac{\ln(S/X) + \left(r - q - \sigma^2/2\right)T}{\sigma\sqrt{T}}.$$

The option price $P_{ij} = P\left(X = X_i, T = T_j\right)$ is computed for each strike price in a set X_i, $i = 1, 2, \ldots, m$, and for each expiry time in a set T_j, $j = 1, 2, \ldots, n$.

4 References

Reiner E and Rubinstein M (1991) Unscrambling the binary code *Risk* **4**

5 Parameters

5.1 Compulsory Input Parameters

1: **calput** – CHARACTER(1)

Determines whether the option is a call or a put.

calput = 'C'
A call; the holder has a right to buy.

calput = 'P'
A put; the holder has a right to sell.

Constraint: **calput** = 'C' or 'P'.

2: **x**(**m**) – REAL (KIND=nag_wp) array

x(i) must contain X_i, the ith strike price, for $i = 1, 2, \ldots, \mathbf{m}$.

Constraint: **x**(i) $\geq z$ and **x**(i) $\leq 1/z$, where $z =$ x02am(), the safe range parameter, for $i = 1, 2, \ldots, \mathbf{m}$.

3: **s** – REAL (KIND=nag_wp)

S, the price of the underlying asset.

Constraint: **s** $\geq z$ and **s** $\leq 1.0/z$, where $z =$ x02am(), the safe range parameter.

4: **k** – REAL (KIND=nag_wp)

The amount, K, to be paid at expiration if the option is in-the-money, i.e., if **s** $>$ **x**(i) when **calput** = 'C', or if **s** $<$ **x**(i) when **calput** = 'P', for $i = 1, 2, \ldots, m$.

Constraint: **k** ≥ 0.0.

5: **t**(**n**) – REAL (KIND=nag_wp) array

t(i) must contain T_i, the ith time, in years, to expiry, for $i = 1, 2, \ldots, \mathbf{n}$.

Constraint: **t**(i) $\geq z$, where $z =$ x02am(), the safe range parameter, for $i = 1, 2, \ldots, \mathbf{n}$.

6: **sigma** – REAL (KIND=nag_wp)

σ, the volatility of the underlying asset. Note that a rate of 15% should be entered as 0.15.

Constraint: **sigma** > 0.0.

7: **r** – REAL (KIND=nag_wp)

r, the annual risk-free interest rate, continuously compounded. Note that a rate of 5% should be entered as 0.05.

Constraint: **r** ≥ 0.0.

8: **q** – REAL (KIND=nag_wp)

q, the annual continuous yield rate. Note that a rate of 8% should be entered as 0.08.

Constraint: **q** ≥ 0.0.

5.2 Optional Input Parameters

1: **m** – INTEGER

Default: the dimension of the array **x**.

The number of strike prices to be used.

Constraint: **m** ≥ 1.

2: **n** – INTEGER

Default: the dimension of the array **t**.

The number of times to expiry to be used.

Constraint: **n** ≥ 1.

5.3 Output Parameters

1: **p**$(ldp, \mathbf{n})$ – REAL (KIND=nag_wp) array

$ldp = \mathbf{m}$.

p(i, j) contains P_{ij}, the option price evaluated for the strike price $\mathbf{x}_i$ at expiry $\mathbf{t}_j$ for $i = 1, 2, \ldots, \mathbf{m}$ and $j = 1, 2, \ldots, \mathbf{n}$.

2: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, **calput** $= \langle value \rangle$ was an illegal value.

ifail $= 2$

Constraint: **m** ≥ 1.

ifail $= 3$

Constraint: **n** ≥ 1.

ifail $= 4$

Constraint: $\mathbf{x}(i) \geq \langle value \rangle$ and $\mathbf{x}(i) \leq \langle value \rangle$.

ifail $= 5$

Constraint: $\mathbf{s} \geq \langle value \rangle$ and $\mathbf{s} \leq \langle value \rangle$.

ifail $= 6$

Constraint: **k** ≥ 0.0.

ifail $= 7$

Constraint: $\mathbf{t}(i) \geq \langle value \rangle$.

ifail $= 8$

Constraint: **sigma** > 0.0.

ifail $= 9$

Constraint: **r** ≥ 0.0.

ifail $= 10$

Constraint: **q** ≥ 0.0.

ifail $= 12$

Constraint: $ldp \geq \mathbf{m}$.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

The accuracy of the output is dependent on the accuracy of the cumulative Normal distribution function, Φ. This is evaluated using a rational Chebyshev expansion, chosen so that the maximum relative error in the expansion is of the order of the ***machine precision*** (see nag_specfun_cdf_normal (s15ab) and nag_specfun_erfc_real (s15ad)). An accuracy close to ***machine precision*** can generally be expected.

8 Further Comments

None.

9 Example

This example computes the price of a cash-or-nothing put with a time to expiry of 0.75 years, a stock price of 100 and a strike price of 80. The risk-free interest rate is 6% per year and the volatility is 35% per year. If the option is in-the-money at expiration, i.e., if $S > X$, the payoff is 10.

9.1 Program Text

```
function s30ca_example

fprintf('s30ca example results\n\n');

put = 'P';
s = 100.0;
k = 10.0;
sigma = 0.35;
r = 0.06;
q = 0.0;
x = [80.0];
t = [0.75];
[p, ifail] = s30ca( ...
                   put, x, s, k, t, sigma, r, q);

fprintf('\nBinary (Digital): Cash-or-Nothing\n European Put :\n');
fprintf('  Spot       =   %9.4f\n', s);
fprintf('  Payout     =   %9.4f\n', k);
fprintf('  Volatility =   %9.4f\n', sigma);
fprintf('  Rate       =   %9.4f\n', r);
fprintf('  Dividend   =   %9.4f\n\n', q);

fprintf('   Strike    Expiry   Option Price\n');
for i=1:1
  for j=1:1
    fprintf('%9.4f %9.4f %9.4f\n', x(i), t(j), p(i,j));
  end
end
```

9.2 Program Results

```
     s30ca example results

Binary (Digital): Cash-or-Nothing
 European Put :
  Spot       =    100.0000
  Payout     =     10.0000
```

```
  Volatility =      0.3500
  Rate       =      0.0600
  Dividend   =      0.0000

   Strike    Expiry   Option Price
  80.0000    0.7500    2.2155
```