NAG Toolbox

nag_specfun_opt_lookback_fls_price (s30ba)

1 Purpose

nag_specfun_opt_lookback_fls_price (s30ba) computes the price of a floating-strike lookback option.

2 Syntax

```
[p, ifail] = nag_specfun_opt_lookback_fls_price(calput, sm, s, t, sigma, r, q,
'm', m, 'n', n)
```

```
[p, ifail] = s30ba(calput, sm, s, t, sigma, r, q, 'm', m, 'n', n)
```

3 Description

nag_specfun_opt_lookback_fls_price (s30ba) computes the price of a floating-strike lookback call or put option. A call option of this type confers the right to buy the underlying asset at the lowest price, S_{min}, observed during the lifetime of the contract. A put option gives the holder the right to sell the underlying asset at the maximum price, S_{max}, observed during the lifetime of the contract. Thus, at expiry, the payoff for a call option is $S - S_{\mathrm{min}}$, and for a put, $S_{\mathrm{max}} - S$.

For a given minimum value the price of a floating-strike lookback call with underlying asset price, S, and time to expiry, T, is

$$P_{\mathrm{call}} = Se^{-qT}\Phi(a_1) - S_{\mathrm{min}}\,e^{-rT}\Phi(a_2) + Se^{-rT}\,\frac{\sigma^2}{2b}\left[\left(\frac{S}{S_{\mathrm{min}}}\right)^{-2b/\sigma^2}\Phi\left(-a_1+\frac{2b}{\sigma}\sqrt{T}\right)-e^{bT}\Phi(-a_1)\right],$$

where $b = r - q \neq 0$. The volatility, σ, risk-free interest rate, r, and annualised dividend yield, q, are constants. When $r = q$, the option price is given by

$$P_{\mathrm{call}} = Se^{-qT}\Phi(a_1) - S_{\mathrm{min}}\,e^{-rT}\Phi(a_2) + Se^{-rT}\sigma\sqrt{T}[\phi(a_1) + a_1(\Phi(a_1) - 1)].$$

The corresponding put price is (for $b \neq 0$),

$$P_{\mathrm{put}} = S_{\mathrm{max}}\,e^{-rT}\Phi(-a_2) - Se^{-qT}\Phi(-a_1) + Se^{-rT}\,\frac{\sigma^2}{2b}\left[-\left(\frac{S}{S_{\mathrm{max}}}\right)^{-2b/\sigma^2}\Phi\left(a_1-\frac{2b}{\sigma}\sqrt{T}\right)+e^{bT}\Phi(a_1)\right].$$

When $r = q$,

$$P_{\mathrm{put}} = S_{\mathrm{max}}\,e^{-rT}\Phi(-a_2) - Se^{-qT}\Phi(-a_1) + Se^{-rT}\sigma\sqrt{T}[\phi(a_1) + a_1\Phi(a_1)].$$

In the above, Φ denotes the cumulative Normal distribution function,

$$\Phi(x) = \int_{-\infty}^{x}\phi(y)dy$$

where ϕ denotes the standard Normal probability density function

$$\phi(y) = \frac{1}{\sqrt{2\pi}}\exp\left(-y^2/2\right)$$

and

$$a_1 = \frac{\ln(S/S_{\mathrm{m}}) + \left(b + \sigma^2/2\right)T}{\sigma\sqrt{T}}$$

$$a_2 = a_1 - \sigma\sqrt{T}$$

where S_m is taken to be the minimum price attained by the underlying asset, S_{min}, for a call and the maximum price, S_{max}, for a put.

The option price $P_{ij} = P(X = X_i, T = T_j)$ is computed for each minimum or maximum observed price in a set $S_{\text{min}}(i)$ or $S_{\text{max}}(i)$, $i = 1, 2, \ldots, m$, and for each expiry time in a set T_j, $j = 1, 2, \ldots, n$.

4 References

Goldman B M, Sosin H B and Gatto M A (1979) Path dependent options: buy at the low, sell at the high *Journal of Finance* **34** 1111–1127

5 Parameters

5.1 Compulsory Input Parameters

1: **calput** – CHARACTER(1)

Determines whether the option is a call or a put.

calput = 'C'
A call; the holder has a right to buy.

calput = 'P'
A put; the holder has a right to sell.

Constraint: **calput** = 'C' or 'P'.

2: **sm**(**m**) – REAL (KIND=nag_wp) array

sm(i) must contain $S_{\text{min}}(i)$, the ith minimum observed price of the underlying asset when **calput** = 'C', or $S_{\text{max}}(i)$, the maximum observed price when **calput** = 'P', for $i = 1, 2, \ldots, \mathbf{m}$.

Constraints:

sm(i) $\geq z$ and **sm**(i) $\leq 1/z$, where $z =$ x02am(), the safe range parameter, for $i = 1, 2, \ldots, \mathbf{m}$;
if **calput** = 'C', **sm**(i) $\leq S$, for $i = 1, 2, \ldots, \mathbf{m}$;
if **calput** = 'P', **sm**(i) $\geq S$, for $i = 1, 2, \ldots, \mathbf{m}$.

3: **s** – REAL (KIND=nag_wp)

S, the price of the underlying asset.

Constraint: **s** $\geq z$ and **s** $\leq 1.0/z$, where $z =$ x02am(), the safe range parameter.

4: **t**(**n**) – REAL (KIND=nag_wp) array

t(i) must contain T_i, the ith time, in years, to expiry, for $i = 1, 2, \ldots, \mathbf{n}$.

Constraint: **t**(i) $\geq z$, where $z =$ x02am(), the safe range parameter, for $i = 1, 2, \ldots, \mathbf{n}$.

5: **sigma** – REAL (KIND=nag_wp)

σ, the volatility of the underlying asset. Note that a rate of 15% should be entered as 0.15.

Constraint: **sigma** > 0.0.

6: **r** – REAL (KIND=nag_wp)

r, the annual risk-free interest rate, continuously compounded. Note that a rate of 5% should be entered as 0.05.

Constraint: **r** ≥ 0.0.

7: **q** – REAL (KIND=nag_wp)

q, the annual continuous yield rate. Note that a rate of 8% should be entered as 0.08.

Constraint: **q** ≥ 0.0.

5.2 Optional Input Parameters

1: **m** – INTEGER

Default: the dimension of the array **sm**.

The number of minimum or maximum prices to be used.

Constraint: **m** ≥ 1.

2: **n** – INTEGER

Default: the dimension of the array **t**.

The number of times to expiry to be used.

Constraint: **n** ≥ 1.

5.3 Output Parameters

1: **p**(ldp, **n**) – REAL (KIND=nag_wp) array

$ldp =$ **m**.

p(i, j) contains P_{ij}, the option price evaluated for the minimum or maximum observed price $S_{\min}(i)$ or $S_{\max}(i)$ at expiry $\mathbf{t}_j$ for $i = 1, 2, \ldots,$ **m** and $j = 1, 2, \ldots,$ **n**.

2: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, **calput** $= \langle$*value*$\rangle$ was an illegal value.

ifail $= 2$

Constraint: **m** ≥ 1.

ifail $= 3$

Constraint: **n** ≥ 1.

ifail $= 4$

Constraint: $\langle$*value*$\rangle \leq$ **sm**$(i) \leq \langle$*value*$\rangle$ for all i.

ifail $= 5$

Constraint: **s** $\geq \langle$*value*$\rangle$ and **s** $\leq \langle$*value*$\rangle$.

ifail $= 6$

Constraint: **t**$(i) \geq \langle$*value*$\rangle$ for all i.

ifail $= 7$

Constraint: **sigma** > 0.0.

ifail $= 8$

Constraint: **r** ≥ 0.0.

ifail $= 9$

Constraint: **q** ≥ 0.0.

ifail $= 11$

Constraint: $ldp \geq$ **m**.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

The accuracy of the output is dependent on the accuracy of the cumulative Normal distribution function, Φ. This is evaluated using a rational Chebyshev expansion, chosen so that the maximum relative error in the expansion is of the order of the ***machine precision*** (see nag_specfun_cdf_normal (s15ab) and nag_specfun_erfc_real (s15ad)). An accuracy close to ***machine precision*** can generally be expected.

8 Further Comments

None.

9 Example

This example computes the price of a floating-strike lookback call with a time to expiry of 6 months and a stock price of 120. The minimum price observed so far is 100. The risk-free interest rate is 10% per year and the volatility is 30% per year with an annual dividend return of 6%.

9.1 Program Text

```
function s30ba_example

fprintf('s30ba example results\n\n');

put = 'c';
s = 120;
sigma = 0.3;
r = 0.1;
q = 0.06;
sm = [100.0];
t = [0.5];

[p, ifail] = s30ba( ...
                    put, sm , s, t, sigma, r, q);

fprintf('\nFloating-strike Lookback\n European Call :\n');
```

```
fprintf('  Spot       =   %9.4f\n', s);
fprintf('  Volatility =   %9.4f\n', sigma);
fprintf('  Rate       =   %9.4f\n', r);
fprintf('  Dividend   =   %9.4f\n\n', q);

fprintf('   Strike    Expiry   Option Price\n');
for i=1:1
  for j=1:1
    fprintf('%9.4f %9.4f %9.4f\n', sm(i), t(j), p(i,j));
  end
end
```

9.2 Program Results

```
     s30ba example results

Floating-strike Lookback
 European Call :
  Spot       =    120.0000
  Volatility =      0.3000
  Rate       =      0.1000
  Dividend   =      0.0600

   Strike    Expiry   Option Price
 100.0000    0.5000   25.3534
```