NAG Toolbox

nag_specfun_ellipint_legendre_3 (s21bg)

1 Purpose

nag_specfun_ellipint_legendre_3 (s21bg) returns a value of the classical (Legendre) form of the incomplete elliptic integral of the third kind, via the function name.

2 Syntax

```
[result, ifail] = nag_specfun_ellipint_legendre_3(dn, phi, dm)
[result, ifail] = s21bg(dn, phi, dm)
```

3 Description

nag_specfun_ellipint_legendre_3 (s21bg) calculates an approximation to the integral

$$\Pi(n;\phi \mid m) = \int_0^\phi \left(1 - n\sin^2\theta\right)^{-1}\left(1 - m\sin^2\theta\right)^{-\frac{1}{2}} d\theta,$$

where $0 \leq \phi \leq \frac{\pi}{2}$, $m\sin^2\phi \leq 1$, m and $\sin\phi$ may not both equal one, and $n\sin^2\phi \neq 1$.

The integral is computed using the symmetrised elliptic integrals of Carlson (Carlson (1979) and Carlson (1988)). The relevant identity is

$$\Pi(n;\phi \mid m) = \sin\phi R_F(q,r,1) + \frac{1}{3}n\sin^3\phi R_J(q,r,1,s),$$

where $q = \cos^2\phi$, $r = 1 - m\sin^2\phi$, $s = 1 - n\sin^2\phi$, R_F is the Carlson symmetrised incomplete elliptic integral of the first kind (see nag_specfun_ellipint_symm_1 (s21bb)) and R_J is the Carlson symmetrised incomplete elliptic integral of the third kind (see nag_specfun_ellipint_symm_3 (s21bd)).

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Carlson B C (1979) Computing elliptic integrals by duplication *Numerische Mathematik* **33** 1–16

Carlson B C (1988) A table of elliptic integrals of the third kind *Math. Comput.* **51** 267–280

5 Parameters

5.1 Compulsory Input Parameters

1: **dn** – REAL (KIND=nag_wp)
2: **phi** – REAL (KIND=nag_wp)
3: **dm** – REAL (KIND=nag_wp)

The arguments n, ϕ and m of the function.

Constraints:

$0.0 \leq$ **phi** $\leq \frac{\pi}{2}$;
dm $\times \sin^2($**phi**$) \leq 1.0$;
Only one of sin(**phi**) and **dm** may be 1.0;
dn $\times \sin^2($**phi**$) \neq 1.0$.

Note that $\mathbf{dm} \times \sin^2(\mathbf{phi}) = 1.0$ is allowable, as long as $\mathbf{dm} \neq 1.0$.

5.2 Optional Input Parameters

None.

5.3 Output Parameters

1: **result**

The result of the function.

2: **ifail** – INTEGER

$\mathbf{ifail} = 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

$\mathbf{ifail} = 1$

Constraint: $0 \leq \mathbf{phi} \leq (\pi/2)$.

$\mathbf{ifail} = 2$

Constraint: $\mathbf{dm} \times \sin^2(\mathbf{phi}) \leq 1.0$.

$\mathbf{ifail} = 3$ (*warning*)

On entry, $\sin(\mathbf{phi}) = 1$ and $\mathbf{dm} = 1.0$; the integral is infinite.

$\mathbf{ifail} = 4$ (*warning*)

Constraint: $\mathbf{dn} \times \sin^2(\mathbf{phi}) \neq 1.0$.

$\mathbf{ifail} = -99$

An unexpected error has been triggered by this routine. Please contact NAG.

$\mathbf{ifail} = -399$

Your licence key may have expired or may not have been installed correctly.

$\mathbf{ifail} = -999$

Dynamic memory allocation failed.

7 Accuracy

In principle nag_specfun_ellipint_legendre_3 (s21bg) is capable of producing full ***machine precision***. However round-off errors in internal arithmetic will result in slight loss of accuracy. This loss should never be excessive as the algorithm does not involve any significant amplification of round-off error. It is reasonable to assume that the result is accurate to within a small multiple of the ***machine precision***.

8 Further Comments

You should consult the S Chapter Introduction, which shows the relationship between this function and the Carlson definitions of the elliptic integrals. In particular, the relationship between the argument-constraints for both forms becomes clear.

For more information on the algorithms used to compute R_F and R_J, see the function documents for nag_specfun_ellipint_symm_1 (s21bb) and nag_specfun_ellipint_symm_3 (s21bd), respectively.

If you wish to input a value of **phi** outside the range allowed by this function you should refer to Section 17.4 of Abramowitz and Stegun (1972) for useful identities.

9 Example

This example simply generates a small set of nonextreme arguments that are used with the function to produce the table of results.

9.1 Program Text

```
function s21bg_example

fprintf('s21bg example results\n\n');

dn  = [0.1   -0.2   0.3];
phi = [pi/6  pi/3   pi/2];
dm  = [1/4   1/2    3/4];
result = phi;

for j = 1:numel(phi)
  [result(j), ifail] = s21bg(dn(j), phi(j), dm(j));
end

fprintf('    n       phi      m       Pi(n;phi|m)\n');
fprintf(' %7.2f %7.2f %7.2f %12.4f\n', [dn; phi; dm; result]);
```

9.2 Program Results

```
s21bg example results

    n       phi      m       Pi(n;phi|m)
    0.10    0.52    0.25       0.5341
   -0.20    1.05    0.50       1.0778
    0.30    1.57    0.75       2.6568
```