NAG Toolbox

nag_specfun_ellipint_symm_1 (s21bb)

1 Purpose

nag_specfun_ellipint_symm_1 (s21bb) returns a value of the symmetrised elliptic integral of the first kind, via the function name.

2 Syntax

```
[result, ifail] = nag_specfun_ellipint_symm_1(x, y, z)
[result, ifail] = s21bb(x, y, z)
```

3 Description

nag_specfun_ellipint_symm_1 (s21bb) calculates an approximation to the integral

$$R_F(x,y,z) = \tfrac{1}{2}\int_0^\infty \frac{dt}{\sqrt{(t+x)(t+y)(t+z)}}$$

where x, y, $z \geq 0$ and at most one is zero.

The basic algorithm, which is due to Carlson (1979) and Carlson (1988), is to reduce the arguments recursively towards their mean by the rule:

$x_0 = \min(x,y,z), \quad z_0 = \max(x,y,z),$

$y_0 =$ remaining third intermediate value argument.

(This ordering, which is possible because of the symmetry of the function, is done for technical reasons related to the avoidance of overflow and underflow.)

$$\begin{array}{lcl} \mu_n & = & (x_n + y_n + z_n)/3 \\ X_n & = & (1 - x_n)/\mu_n \\ Y_n & = & (1 - y_n)/\mu_n \\ Z_n & = & (1 - z_n)/\mu_n \\ \lambda_n & = & \sqrt{x_n y_n} + \sqrt{y_n z_n} + \sqrt{z_n x_n} \\ x_{n+1} & = & (x_n + \lambda_n)/4 \\ y_{n+1} & = & (y_n + \lambda_n)/4 \\ z_{n+1} & = & (z_n + \lambda_n)/4 \end{array}$$

$\epsilon_n = \max(|X_n|, |Y_n|, |Z_n|)$ and the function may be approximated adequately by a fifth order power series:

$$R_F(x,y,z) = \frac{1}{\sqrt{\mu_n}}\left(1 - \frac{E_2}{10} + \frac{E_2^2}{24} - \frac{3E_2E_3}{44} + \frac{E_3}{14}\right)$$

where $E_2 = X_nY_n + Y_nZ_n + Z_nX_n$, $E_3 = X_nY_nZ_n$.

The truncation error involved in using this approximation is bounded by $\epsilon_n^6/4(1-\epsilon_n)$ and the recursive process is stopped when this truncation error is negligible compared with the ***machine precision***.

Within the domain of definition, the function value is itself representable for all representable values of its arguments. However, for values of the arguments near the extremes the above algorithm must be modified so as to avoid causing underflows or overflows in intermediate steps. In extreme regions arguments are prescaled away from the extremes and compensating scaling of the result is done before returning to the calling program.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Carlson B C (1979) Computing elliptic integrals by duplication *Numerische Mathematik* **33** 1–16

Carlson B C (1988) A table of elliptic integrals of the third kind *Math. Comput.* **51** 267–280

5 Parameters

5.1 Compulsory Input Parameters

1: **x** – REAL (KIND=nag_wp)
2: **y** – REAL (KIND=nag_wp)
3: **z** – REAL (KIND=nag_wp)

The arguments x, y and z of the function.

Constraint: **x**, **y**, **z** ≥ 0.0 and only one of **x**, **y** and **z** may be zero.

5.2 Optional Input Parameters

None.

5.3 Output Parameters

1: **result**

The result of the function.

2: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, one or more of **x**, **y** and **z** is negative; the function is undefined.

ifail $= 2$

On entry, two or more of **x**, **y** and **z** are zero; the function is undefined. On softfailure, the function returns zero.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

In principle nag_specfun_ellipint_symm_1 (s21bb) is capable of producing full ***machine precision***. However round-off errors in internal arithmetic will result in slight loss of accuracy. This loss should never be excessive as the algorithm does not involve any significant amplification of round-off error. It is reasonable to assume that the result is accurate to within a small multiple of the ***machine precision***.

8 Further Comments

You should consult the S Chapter Introduction which shows the relationship of this function to the classical definitions of the elliptic integrals.

If two arguments are equal, the function reduces to the elementary integral R_C, computed by nag_specfun_ellipint_symm_1_degen (s21ba).

9 Example

This example simply generates a small set of nonextreme arguments which are used with the function to produce the table of low accuracy results.

9.1 Program Text

```
     function s21bb_example

fprintf('s21bb example results\n\n');

x = [0.5   1   1.5];
y = x + 0.5;
z = y + 0.5;
result = x;

for j=1:numel(x)
  [result(j), ifail] = s21bb(x(j), y(j), z(j));
end

fprintf('    x      y      z      R_F(x,y,z)\n');
fprintf('%7.2f%7.2f%7.2f%12.4f\n',[x; y; z; result]);
```

9.2 Program Results

```
     s21bb example results

    x      y      z      R_F(x,y,z)
   0.50   1.00   1.50      1.0281
   1.00   1.50   2.00      0.8260
   1.50   2.00   2.50      0.7116
```