

NAG Toolbox

nag_specfun_fresnel_c (s20ad)

1 Purpose

nag_specfun_fresnel_c (s20ad) returns a value for the Fresnel integral $C(x)$, via the function name.

2 Syntax

```
[result, ifail] = nag_specfun_fresnel_c(x)
[result, ifail] = s20ad(x)
```

3 Description

nag_specfun_fresnel_c (s20ad) evaluates an approximation to the Fresnel integral

$$C(x) = \int_0^x \cos\left(\frac{\pi}{2}t^2\right) dt.$$

Note: $C(x) = -C(-x)$, so the approximation need only consider $x \geq 0.0$.

The function is based on three Chebyshev expansions:

For $0 < x \leq 3$,

$$C(x) = x \sum_{r=0} a_r T_r(t), \quad \text{with } t = 2\left(\frac{x}{3}\right)^4 - 1.$$

For $x > 3$,

$$C(x) = \frac{1}{2} + \frac{f(x)}{x} \sin\left(\frac{\pi}{2}x^2\right) - \frac{g(x)}{x^3} \cos\left(\frac{\pi}{2}x^2\right),$$

where $f(x) = \sum_{r=0} b_r T_r(t)$,

and $g(x) = \sum_{r=0} c_r T_r(t)$,

with $t = 2\left(\frac{3}{x}\right)^4 - 1$.

For small x , $C(x) \simeq x$. This approximation is used when x is sufficiently small for the result to be correct to **machine precision**.

For large x , $f(x) \simeq \frac{1}{\pi}$ and $g(x) \simeq \frac{1}{\pi^2}$. Therefore for moderately large x , when $\frac{1}{\pi^2 x^3}$ is negligible compared with $\frac{1}{2}$, the second term in the approximation for $x > 3$ may be dropped. For very large x , when $\frac{1}{\pi x}$ becomes negligible, $C(x) \simeq \frac{1}{2}$. However there will be considerable difficulties in calculating $\sin\left(\frac{\pi}{2}x^2\right)$ accurately before this final limiting value can be used. Since $\sin\left(\frac{\pi}{2}x^2\right)$ is periodic, its value is essentially determined by the fractional part of x^2 . If $x^2 = N + \theta$, where N is an integer and $0 \leq \theta < 1$, then $\sin\left(\frac{\pi}{2}x^2\right)$ depends on θ and on N modulo 4. By exploiting this fact, it is possible to retain some significance in the calculation of $\sin\left(\frac{\pi}{2}x^2\right)$ either all the way to the very large x limit, or at least until the integer part of $\frac{x}{2}$ is equal to the maximum integer allowed on the machine.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

5.1 Compulsory Input Parameters

- 1: **x** – REAL (KIND=nag_wp)
The argument x of the function.

5.2 Optional Input Parameters

None.

5.3 Output Parameters

- 1: **result**
The result of the function.
- 2: **ifail** – INTEGER
ifail = 0 unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

There are no failure exits from nag_specfun_fresnel_c (s20ad). The argument **ifail** has been included for consistency with other functions in this chapter.

7 Accuracy

Let δ and ϵ be the relative errors in the argument and result respectively.

If δ is somewhat larger than the *machine precision* (i.e. if δ is due to data errors etc.), then ϵ and δ are approximately related by:

$$\epsilon \simeq \left| \frac{x \cos\left(\frac{\pi}{2}x^2\right)}{C(x)} \right| \delta.$$

Figure 1 shows the behaviour of the error amplification factor $\left| \frac{x \cos\left(\frac{\pi}{2}x^2\right)}{C(x)} \right|$.

However, if δ is of the same order as the *machine precision*, then rounding errors could make ϵ slightly larger than the above relation predicts.

For small x , $\epsilon \simeq \delta$ and there is no amplification of relative error.

For moderately large values of x ,

$$|\epsilon| \simeq \left| 2x \cos\left(\frac{\pi}{2}x^2\right) \right| |\delta|$$

and the result will be subject to increasingly large amplification of errors. However the above relation breaks down for large values of x (i.e., when $\frac{1}{x^2}$ is of the order of the *machine precision*); in this region the relative error in the result is essentially bounded by $\frac{2}{\pi x}$.

Hence the effects of error amplification are limited and at worst the relative error loss should not exceed half the possible number of significant figures.

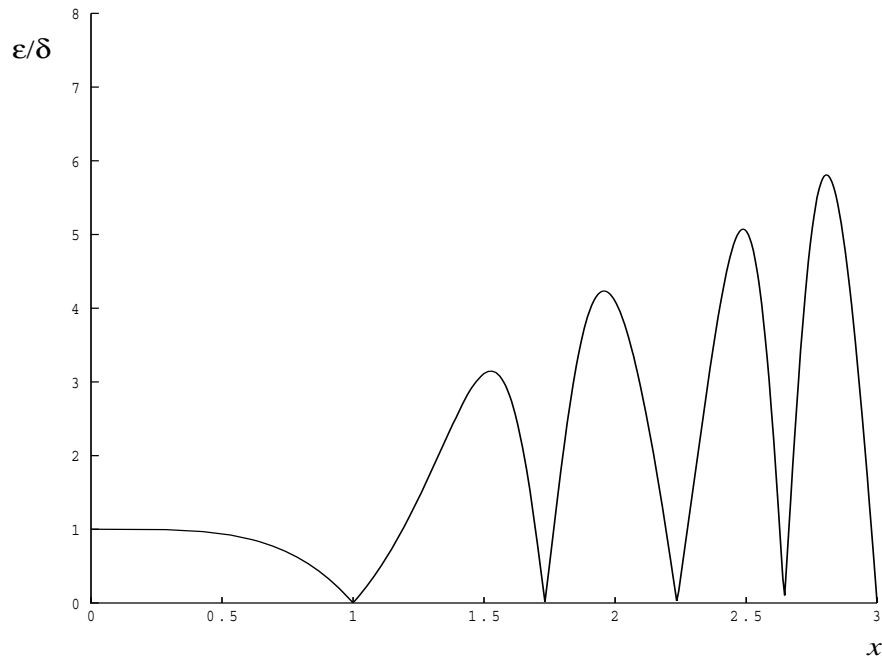


Figure 1

8 Further Comments

None.

9 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

```
function s20ad_example
    fprintf('s20ad example results\n\n');
    x = [0 0.5 1 2 4 5 6 8 10 -1 1000];
    n = size(x,2);
    result = x;
    for j=1:n
        [result(j), ifail] = s20ad(x(j));
    end
    disp('      x      C(x)');
    fprintf('%12.3e%12.3e\n',[x; result]);
    s20ad_plot;
    function s20ad_plot
        x = [-10:0.02:10];
        for j = 1:numel(x)
            [C(j), ifail] = s20ad(x(j));
        end
        fig1 = figure;
        plot(x,C,'-r');
```

```
xlabel('x');  
ylabel('C(x)');  
title('Fresnel Integral C(x)');  
  
% print(fig1,'-dpng','-r75','s20ad_fig1.png');  
% print(fig1,'-deps','-r75','s20ad_fig1.eps');
```

9.2 Program Results

s20ad example results

x	C(x)
0.000e+00	0.000e+00
5.000e-01	4.923e-01
1.000e+00	7.799e-01
2.000e+00	4.883e-01
4.000e+00	4.984e-01
5.000e+00	5.636e-01
6.000e+00	4.995e-01
8.000e+00	4.998e-01
1.000e+01	4.999e-01
-1.000e+00	-7.799e-01
1.000e+03	5.000e-01

