NAG Toolbox

nag_specfun_bessel_zeros (s17al)

1 Purpose

nag_specfun_bessel_zeros (s17al) determines the leading **n** zeros of one of the Bessel functions $J_\alpha(x)$, $Y_\alpha(x)$, $J'_\alpha(x)$ or $Y'_\alpha(x)$ for real x and non-negative α.

2 Syntax

```
[x, ifail] = nag_specfun_bessel_zeros(a, n, mode, 'rel', rel)
[x, ifail] = s17al(a, n, mode, 'rel', rel)
```

3 Description

nag_specfun_bessel_zeros (s17al) attempts to find the leading N zeros of one of the Bessel functions $J_\alpha(x)$, $Y_\alpha(x)$, $J'_\alpha(x)$ or $Y'_\alpha(x)$, where x is real. When α is real, these functions each have an infinite number of real zeros, all of which are simple with the possible exception of $x = 0$. If $\alpha \geq 0$, the nth positive zero is denoted by $j_{\alpha,n}, j'_{\alpha,n}, y_{\alpha,n}$ and $y'_{\alpha,n}$, respectively, for $n = 1, 2, \ldots, N$, except that $x = 0$ is counted as the first zero of $J'_\alpha(x)$ when $\alpha = 0$. Since $J'_0(x) = -J_1(x)$, it therefore follows that $j'_{0,1} = 0$ and $j'_{0,n} = -j_{1,n-1}$ for $n = 2, 3, \ldots, N - 1$. Further details can be found in Section 9.5 of Abramowitz and Stegun (1972).

nag_specfun_bessel_zeros (s17al) is based on Algol 60 procedures given by Temme (1979). Initial approximations to the zeros are computed from asymptotic expansions. These are then improved by higher-order Newton iteration making use of the differential equation for the Bessel functions.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Temme N M (1976) On the numerical evaluation of the ordinary Bessel function of the second kind *J. Comput. Phys.* **21** 343–350

Temme N M (1979) An algorithm with Algol 60 program for the computation of the zeros of ordinary Bessel functions and those of their derivatives *J. Comput. Phys.* **32** 270–279

5 Parameters

5.1 Compulsory Input Parameters

1: **a** – REAL (KIND=nag_wp)

The order α of the function.

Constraint: $0.0 \leq$ **a** ≤ 100000.0.

2: **n** – INTEGER

The number N of zeros required.

Constraint: **n** ≥ 1.

3: **mode** – INTEGER

Specifies the form of the function whose zeros are required.

mode $= 1$
The zeros of $J_\alpha(x)$ are required.

mode $= 2$
The zeros of $Y_\alpha(x)$ are required;

mode $= 3$
The zeros of $J'_\alpha(x)$ are required;

mode $= 4$
The zeros of $Y'_\alpha(x)$ are required.

Constraint: $1 \leq$ **mode** ≤ 4.

5.2 Optional Input Parameters

1: **rel** – REAL (KIND=nag_wp)

Suggested value: the square root of the ***machine precision***.

Default: $\sqrt{\textit{\textbf{machine precision}}}$

The relative accuracy to which the zeros are required.

Constraint: **rel** > 0.0.

5.3 Output Parameters

1: **x**(**n**) – REAL (KIND=nag_wp) array

The N required zeros of the function specified by **mode**.

2: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, **a** < 0.0,
or **a** > 100000.0,
or **n** ≤ 0,
or **mode** < 1,
or **mode** > 4,
or **rel** ≤ 0.0.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

If the value of **rel** is set to 10^{-d}, then the required zeros should have approximately d correct significant digits.

8 Further Comments

None.

9 Example

This example determines the leading five positive zeros of the Bessel function $J_0(x)$.

9.1 Program Text

```
function s17al_example

fprintf('s17al example results\n\n');

a = 0;
n = nag_int(5);
mode = nag_int(1);

[result, ifail] = s17al(a, n, mode);

fprintf('Leading %2d zeros of J_0(x)\n',n);
fprintf('%12.4f\n',result);
```

9.2 Program Results

```
s17al example results

Leading  5 zeros of J_0(x)
      2.4048
      5.5201
      8.6537
     11.7915
     14.9309
```