NAG Toolbox

nag_specfun_bessel_y0_real (s17ac)

1 Purpose

nag_specfun_bessel_y0_real (s17ac) returns the value of the Bessel function $Y_0(x)$, via the function name.

2 Syntax

```
[result, ifail] = nag_specfun_bessel_y0_real(x)
[result, ifail] = s17ac(x)
```

3 Description

nag_specfun_bessel_y0_real (s17ac) evaluates an approximation to the Bessel function of the second kind $Y_0(x)$.

Note: $Y_0(x)$ is undefined for $x \leq 0$ and the function will fail for such arguments.

The function is based on four Chebyshev expansions:

For $0 < x \leq 8$,

$$Y_0(x) = \frac{2}{\pi} \ln x \sum_{r=0}{}' a_r T_r(t) + \sum_{r=0}{}' b_r T_r(t), \qquad \text{with } t = 2\left(\frac{x}{8}\right)^2 - 1.$$

For $x > 8$,

$$Y_0(x) = \sqrt{\frac{2}{\pi x}} \left\{ P_0(x) \sin\left(x - \frac{\pi}{4}\right) + Q_0(x) \cos\left(x - \frac{\pi}{4}\right) \right\}$$

where $P_0(x) = \sum_{r=0} c_r T_r(t)$,

and $Q_0(x) = \frac{8}{x} \sum_{r=0} d_r T_r(t)$, with $t = 2\left(\frac{8}{x}\right)^2 - 1$.

For x near zero, $Y_0(x) \simeq \frac{2}{\pi}\left(\ln\left(\frac{x}{2}\right) + \gamma\right)$, where γ denotes Euler's constant. This approximation is used when x is sufficiently small for the result to be correct to ***machine precision***.

For very large x, it becomes impossible to provide results with any reasonable accuracy (see Section 7), hence the function fails. Such arguments contain insufficient information to determine the phase of oscillation of $Y_0(x)$; only the amplitude, $\sqrt{\frac{2}{\pi n}}$, can be determined and this is returned on softfailure. The range for which this occurs is roughly related to ***machine precision***; the function will fail if $x \gtrsim 1/$***machine precision***.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Clenshaw C W (1962) Chebyshev Series for Mathematical Functions *Mathematical tables* HMSO

5 Parameters

5.1 Compulsory Input Parameters

1: **x** – REAL (KIND=nag_wp)

The argument x of the function.

Constraint: **x** > 0.0.

5.2 Optional Input Parameters

None.

5.3 Output Parameters

1: **result**

The result of the function.

2: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

x is too large. On softfailure the function returns the amplitude of the Y_0 oscillation, $\sqrt{2/(\pi x)}$.

ifail $= 2$

x ≤ 0.0, Y_0 is undefined. On softfailure the function returns zero.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

Let δ be the relative error in the argument and E be the absolute error in the result. (Since $Y_0(x)$ oscillates about zero, absolute error and not relative error is significant, except for very small x.)

If δ is somewhat larger than the machine representation error (e.g., if δ is due to data errors etc.), then E and δ are approximately related by

$$E \simeq |xY_1(x)|\delta$$

(provided E is also within machine bounds). Figure 1 displays the behaviour of the amplification factor $|xY_1(x)|$.

However, if δ is of the same order as the machine representation errors, then rounding errors could make E slightly larger than the above relation predicts.

For very small x, the errors are essentially independent of δ and the function should provide relative accuracy bounded by the ***machine precision***.

For very large x, the above relation ceases to apply. In this region, $Y_0(x) \simeq \sqrt{\frac{2}{\pi x}} \sin\left(x - \frac{\pi}{4}\right)$. The amplitude $\sqrt{\frac{2}{\pi x}}$ can be calculated with reasonable accuracy for all x, but $\sin\left(x - \frac{\pi}{4}\right)$ cannot. If $x - \frac{\pi}{4}$ is written as $2N\pi + \theta$ where N is an integer and $0 \leq \theta < 2\pi$, then $\sin\left(x - \frac{\pi}{4}\right)$ is determined by θ only. If $x \gtrsim \delta^{-1}$, θ cannot be determined with any accuracy at all. Thus if x is greater than, or of the order of the inverse of ***machine precision***, it is impossible to calculate the phase of $Y_0(x)$ and the function must fail.

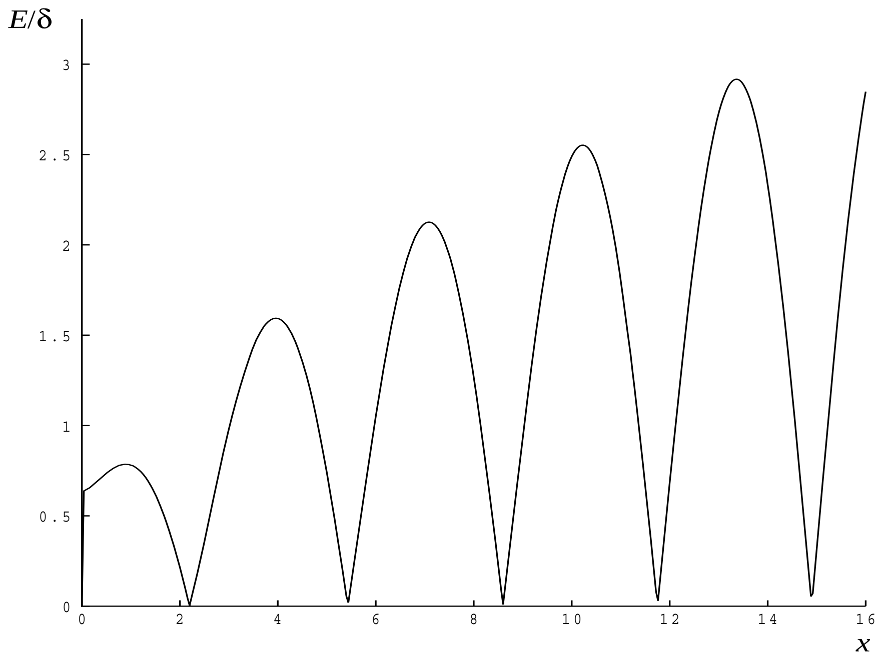


Figure 1

8 Further Comments

None.

9 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

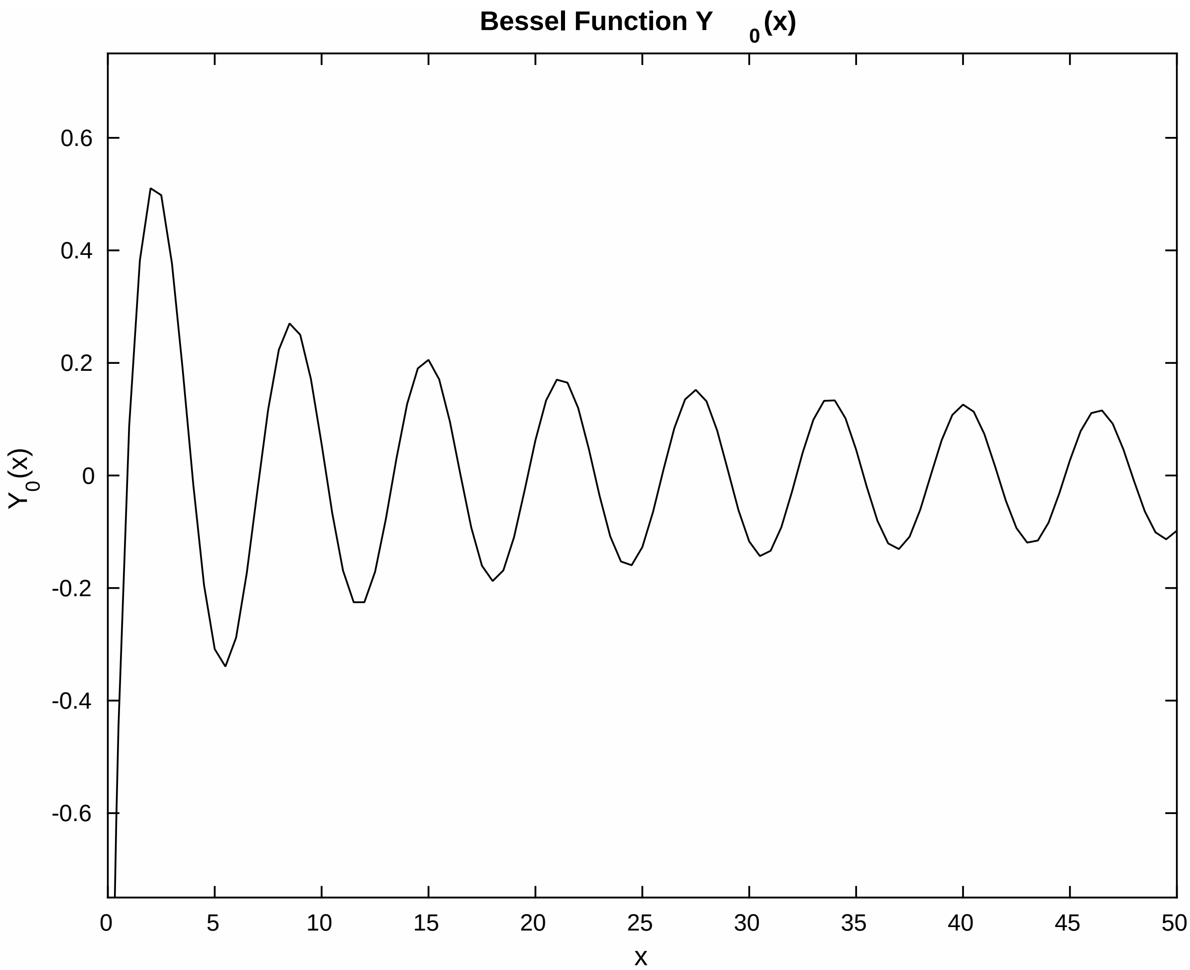
9.1 Program Text

```
     function s17ac_example

fprintf('s17ac example results\n\n');

x = [0.5     1    3    6   8    10   100];
n = size(x,2);
result = x;

for j=1:n
  [result(j), ifail] = s17ac(x(j));
end

disp('      x          Y_0(x)');
fprintf('%12.3e%12.3e\n',[x; result]);
```

```
s17ac_plot;

function s17ac_plot
x = [0.1:0.1:0.4,0.5:0.5:50];
  for j = 1:numel(x)
    [Y0(j), ifail] = s17ac(x(j));
  end

  fig1 = figure;
  plot(x,Y0,'-r');
  xlabel('x');
  ylabel('Y_0(x)');
  title('Bessel Function Y_0(x)');
  axis([0 50 -0.75 0.75]);

  % print(fig1,'-dpng','-r75','s17ac_fig1.png');
  % print(fig1,'-deps','-r75','s17ac_fig1.eps');
```

9.2 Program Results

```
     s17ac example results

      x          Y_0(x)
   5.000e-01  -4.445e-01
   1.000e+00   8.826e-02
   3.000e+00   3.769e-01
   6.000e+00  -2.882e-01
   8.000e+00   2.235e-01
   1.000e+01   5.567e-02
   1.000e+02  -7.724e-02
```


Bessel Function Y0(x)
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
Y0(x)
0
5
10
15
20
25
30
35
40
45
50
x