

NAG Toolbox

nag_specfun_erfc_real (s15ad)

1 Purpose

nag_specfun_erfc_real (s15ad) returns the value of the complementary error function, $\operatorname{erfc}(x)$, via the function name.

2 Syntax

```
[result, ifail] = nag_specfun_erfc_real(x)
[result, ifail] = s15ad(x)
```

3 Description

nag_specfun_erfc_real (s15ad) calculates an approximate value for the complement of the error function

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-t^2} dt = 1 - \operatorname{erf}(x).$$

Let \hat{x} be the root of the equation $\operatorname{erfc}(x) - \operatorname{erf}(x) = 0$ (then $\hat{x} \approx 0.46875$). For $|x| \leq \hat{x}$ the value of $\operatorname{erfc}(x)$ is based on the following rational Chebyshev expansion for $\operatorname{erf}(x)$:

$$\operatorname{erf}(x) \approx x R_{\ell,m}(x^2),$$

where $R_{\ell,m}$ denotes a rational function of degree ℓ in the numerator and m in the denominator.

For $|x| > \hat{x}$ the value of $\operatorname{erfc}(x)$ is based on a rational Chebyshev expansion for $\operatorname{erfc}(x)$: for $\hat{x} < |x| \leq 4$ the value is based on the expansion

$$\operatorname{erfc}(x) \approx e^{x^2} R_{\ell,m}(x);$$

and for $|x| > 4$ it is based on the expansion

$$\operatorname{erfc}(x) \approx \frac{e^{x^2}}{x} \left(\frac{1}{\sqrt{\pi}} + \frac{1}{x^2} R_{\ell,m}(1/x^2) \right).$$

For each expansion, the specific values of ℓ and m are selected to be minimal such that the maximum relative error in the expansion is of the order 10^{-d} , where d is the maximum number of decimal digits that can be accurately represented for the particular implementation (see nag_machine_decimal_digits (x02be)).

For $|x| \geq x_{\text{hi}}$ there is a danger of setting underflow in $\operatorname{erfc}(x)$. For $x \geq x_{\text{hi}}$, nag_specfun_erfc_real (s15ad) returns $\operatorname{erfc}(x) = 0$; for $x \leq -x_{\text{hi}}$ it returns $\operatorname{erfc}(x) = 2$.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Cody W J (1969) Rational Chebyshev approximations for the error function *Math.Comp.* **23** 631–637

5 Parameters

5.1 Compulsory Input Parameters

- 1: **x** – REAL (KIND=nag_wp)
The argument x of the function.

5.2 Optional Input Parameters

None.

5.3 Output Parameters

- 1: **result**
The result of the function.
- 2: **ifail** – INTEGER
ifail = 0 unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

There are no failure exits from nag_specfun_erfc_real (s15ad). The argument **ifail** has been included for consistency with other functions in this chapter.

7 Accuracy

If δ and ϵ are relative errors in the argument and result, respectively, then in principle

$$|\epsilon| \simeq \left| \frac{2xe^{-x^2}}{\sqrt{\pi} \operatorname{erfc}(x)} \delta \right|.$$

That is, the relative error in the argument, x , is amplified by a factor $\frac{2xe^{-x^2}}{\sqrt{\pi} \operatorname{erfc}(x)}$ in the result.

The behaviour of this factor is shown in Figure 1.

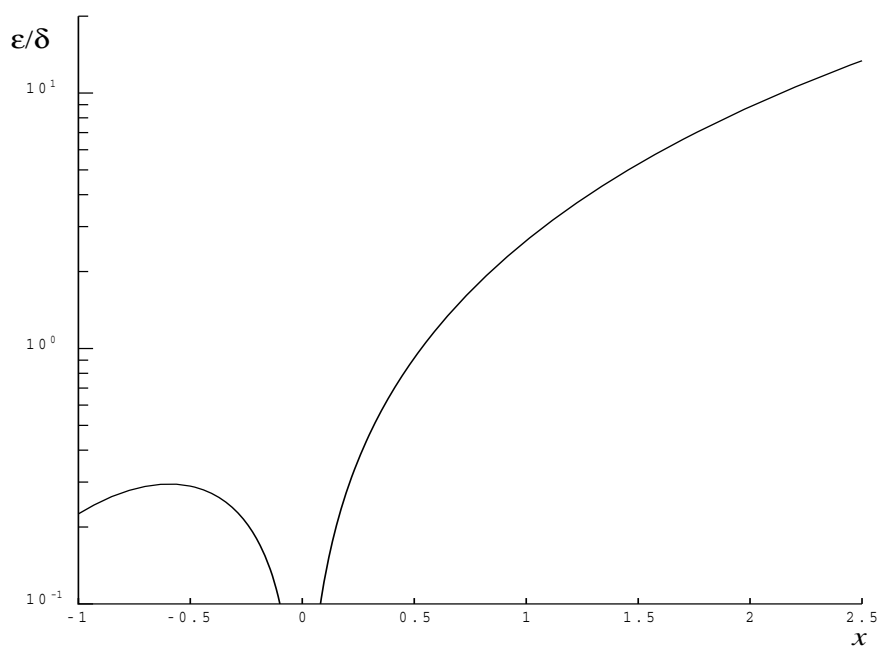


Figure 1

It should be noted that near $x = 0$ this factor behaves as $\frac{2x}{\sqrt{\pi}}$ and hence the accuracy is largely determined by the *machine precision*. Also for large negative x , where the factor is $\sim \frac{xe^{-x^2}}{\sqrt{\pi}}$, accuracy is mainly limited by *machine precision*. However, for large positive x , the factor becomes $\sim 2x^2$ and to an extent relative accuracy is necessarily lost. The absolute accuracy E is given by

$$E \simeq \frac{2xe^{-x^2}}{\sqrt{\pi}}\delta$$

so absolute accuracy is guaranteed for all x .

8 Further Comments

None.

9 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

```
function s15ad_example

fprintf('s15ad example results\n\n');

x = [-10   -1     0     1     10];
n = size(x,2);
result = x;

for j=1:n
    [result(j), ifail] = s15ad(x(j));
end

disp('      x      erfc(x)');
fprintf('%12.3e%12.3e\n',[x; result]);
```

9.2 Program Results

```
s15ad example results

      x      erfc(x)
-1.000e+01  2.000e+00
-1.000e+00  1.843e+00
 0.000e+00  1.000e+00
 1.000e+00  1.573e-01
 1.000e+01  2.088e-45
```
