NAG Toolbox

nag_specfun_cdf_normal (s15ab)

1 Purpose

nag_specfun_cdf_normal (s15ab) returns the value of the cumulative Normal distribution function, $P(x)$, via the function name.

2 Syntax

```
[result, ifail] = nag_specfun_cdf_normal(x)
[result, ifail] = s15ab(x)
```

3 Description

nag_specfun_cdf_normal (s15ab) evaluates an approximate value for the cumulative Normal distribution function

$$P(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-u^2/2}\, du.$$

The function is based on the fact that

$$P(x) = \tfrac{1}{2}\mathrm{erfc}\left(\frac{-x}{\sqrt{2}}\right)$$

and it calls nag_specfun_erfc_real (s15ad) to obtain a value of $erfc$ for the appropriate argument.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

5.1 Compulsory Input Parameters

1: **x** – REAL (KIND=nag_wp)

The argument x of the function.

5.2 Optional Input Parameters

None.

5.3 Output Parameters

1: **result**

The result of the function.

2: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

There are no failure exits from this function. The argument **ifail** is included for consistency with other functions in this chapter.

7 Accuracy

Because of its close relationship with $erfc$, the accuracy of this function is very similar to that in nag_specfun_erfc_real (s15ad). If ϵ and δ are the relative errors in result and argument, respectively, they are in principle related by

$$|\epsilon| \simeq \left| \frac{xe^{-\frac{1}{2}x^2}}{\sqrt{2\pi}P(x)} \delta \right|$$

so that the relative error in the argument, x, is amplified by a factor, $\frac{xe^{-\frac{1}{2}x^2}}{\sqrt{2\pi}P(x)}$, in the result.

For x small and for x positive this factor is always less than one and accuracy is mainly limited by ***machine precision***.

For large negative x the factor behaves like $\sim x^2$ and hence to a certain extent relative accuracy is unavoidably lost.

However the absolute error in the result, E, is given by

$$|E| \simeq \left| \frac{xe^{-\frac{1}{2}x^2}}{\sqrt{2\pi}} \delta \right|$$

so absolute accuracy can be guaranteed for all x.

8 Further Comments

None.

9 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

```
     function s15ab_example

fprintf('s15ab example results\n\n');

x = [-20   -1     0    1    2    20];
n = size(x,2);
result = x;

for j=1:n
  [result(j), ifail] = s15ab(x(j));
end

disp('      x          P(x)');
fprintf('%12.3e%12.3e\n',[x; result]);
```

9.2 Program Results

```
s15ab example results

   x           P(x)
-2.000e+01   2.754e-89
-1.000e+00   1.587e-01
 0.000e+00   5.000e-01
 1.000e+00   8.413e-01
 2.000e+00   9.772e-01
 2.000e+01   1.000e+00
```