NAG Toolbox

nag_specfun_gamma_incomplete (s14ba)

1 Purpose

nag_specfun_gamma_incomplete (s14ba) computes values for the incomplete gamma functions $P(a,x)$ and $Q(a,x)$.

2 Syntax

```
[p, q, ifail] = nag_specfun_gamma_incomplete(a, x, tol)
[p, q, ifail] = s14ba(a, x, tol)
```

3 Description

nag_specfun_gamma_incomplete (s14ba) evaluates the incomplete gamma functions in the normalized form

$$P(a,x) = \frac{1}{\Gamma(a)}\int_0^x t^{a-1}e^{-t}\,dt,$$

$$Q(a,x) = \frac{1}{\Gamma(a)}\int_x^\infty t^{a-1}e^{-t}\,dt,$$

with $x \geq 0$ and $a > 0$, to a user-specified accuracy. With this normalization, $P(a,x) + Q(a,x) = 1$.

Several methods are used to evaluate the functions depending on the arguments a and x, the methods including Taylor expansion for $P(a,x)$, Legendre's continued fraction for $Q(a,x)$, and power series for $Q(a,x)$. When both a and x are large, and $a \simeq x$, the uniform asymptotic expansion of Temme (1987) is employed for greater efficiency – specifically, this expansion is used when $a \geq 20$ and $0.7a \leq x \leq 1.4a$.

Once either P or Q is computed, the other is obtained by subtraction from 1. In order to avoid loss of relative precision in this subtraction, the smaller of P and Q is computed first.

This function is derived from the function GAM in Gautschi (1979b).

4 References

Gautschi W (1979a) A computational procedure for incomplete gamma functions *ACM Trans. Math. Software* **5** 466–481

Gautschi W (1979b) Algorithm 542: Incomplete gamma functions *ACM Trans. Math. Software* **5** 482–489

Temme N M (1987) On the computation of the incomplete gamma functions for large values of the parameters *Algorithms for Approximation* (eds J C Mason and M G Cox) Oxford University Press

5 Parameters

5.1 Compulsory Input Parameters

1: **a** – REAL (KIND=nag_wp)

The argument a of the functions.

Constraint: **a** > 0.0.

2: **x** – REAL (KIND=nag_wp)

The argument x of the functions.

Constraint: **x** ≥ 0.0.

3: **tol** – REAL (KIND=nag_wp)

The relative accuracy required by you in the results. If nag_specfun_gamma_incomplete (s14ba) is entered with **tol** greater than 1.0 or less than ***machine precision***, then the value of ***machine precision*** is used instead.

5.2 Optional Input Parameters

None.

5.3 Output Parameters

1: **p** – REAL (KIND=nag_wp)
2: **q** – REAL (KIND=nag_wp)

The values of the functions $P(a,x)$ and $Q(a,x)$ respectively.

3: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, **a** ≤ 0.0.

ifail $= 2$

On entry, **x** < 0.0.

ifail $= 3$

Convergence of the Taylor series or Legendre continued fraction fails within 600 iterations. This error is extremely unlikely to occur; if it does, contact NAG.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

There are rare occasions when the relative accuracy attained is somewhat less than that specified by argument **tol**. However, the error should never exceed more than one or two decimal places. Note also that there is a limit of 18 decimal places on the achievable accuracy, because constants in the function are given to this precision.

8 Further Comments

The time taken for a call of nag_specfun_gamma_incomplete (s14ba) depends on the precision requested through **tol**, and also varies slightly with the input arguments a and x.

9 Example

This example reads values of the argument a and x from a file, evaluates the function and prints the results.

9.1 Program Text

```
     function s14ba_example

fprintf('s14ba example results\n\n');

a = [ 2  7   0.5  20  21];
x = [ 3  1  99.0  21  20];
n = size(x,2);
p = x;
q = x;
tol = x02aj;

for j=1:n
  [p(j), q(j), ifail] = s14ba(a(j), x(j), tol);
end

disp('     a      x     P(a,x)    Q(a,x)');
fprintf('%7.1f%7.1f%10.4f%10.4f\n',[a; x; p; q]);

s14ba_plot;

function s14ba_plot
  x = [0:0.5:20];
  a = [0.1:0.1:0.4,0.5:0.5:20];
  p = zeros(numel(a),numel(x));
  q = p;

  tol = x02aj;
  for i=1:numel(a)
    for j=1:numel(x)
      [p(i,j), q(i,j), ifail] = s14ba(a(i), x(j), tol);
    end
  end

  fig1 = figure;
  hold on
  mesh(x, a, p, 'FaceColor','r');
  mesh(x, a, q, 'FaceColor','g');
  xlabel('x');
  ylabel('a');
  title('Incomplete Gamma Functions');
  legend('P(a,x)','Q(a,x)');
  view(-35, 46);
  hold off;

  % print(fig1,'-dpng','-r75','s14ba_fig1.png');
  % print(fig1,'-deps','-r75','s14ba_fig1.eps');
```

9.2 Program Results

```
s14ba example results

    a       x      P(a,x)     Q(a,x)
  2.0     3.0     0.8009     0.1991
  7.0     1.0     0.0001     0.9999
  0.5    99.0     1.0000     0.0000
 20.0    21.0     0.6157     0.3843
 21.0    20.0     0.4409     0.5591
```