NAG Toolbox

nag_specfun_psi_deriv_real (s14ae)

1 Purpose

nag_specfun_psi_deriv_real (s14ae) returns the value of the kth derivative of the psi function $\psi(x)$ for real x and $k = 0, 1, \ldots, 6$, via the function name.

2 Syntax

```
[result, ifail] = nag_specfun_psi_deriv_real(x, k)
[result, ifail] = s14ae(x, k)
```

3 Description

nag_specfun_psi_deriv_real (s14ae) evaluates an approximation to the kth derivative of the psi function $\psi(x)$ given by

$$\psi^{(k)}(x) = \frac{d^k}{dx^k}\psi(x) = \frac{d^k}{dx^k}\left(\frac{d}{dx}\log_e \Gamma(x)\right),$$

where x is real with $x \neq 0, -1, -2, \ldots$ and $k = 0, 1, \ldots, 6$. For negative noninteger values of x, the recurrence relationship

$$\psi^{(k)}(x+1) = \psi^{(k)}(x) + \frac{d^k}{dx^k}\left(\frac{1}{x}\right)$$

is used. The value of $\dfrac{(-1)^{k+1}\psi^{(k)}(x)}{k!}$ is obtained by a call to nag_specfun_polygamma_deriv (s14ad), which is based on the function PSIFN in Amos (1983).

Note that $\psi^{(k)}(x)$ is also known as the *polygamma* function. Specifically, $\psi^{(0)}(x)$ is often referred to as the *digamma* function and $\psi^{(1)}(x)$ as the *trigamma* function in the literature. Further details can be found in Abramowitz and Stegun (1972).

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Amos D E (1983) Algorithm 610: A portable FORTRAN subroutine for derivatives of the psi function *ACM Trans. Math. Software* **9** 494–502

5 Parameters

5.1 Compulsory Input Parameters

1: **x** – REAL (KIND=nag_wp)

The argument x of the function.

Constraint: **x** must not be ‘too close’ (see Section 6) to a non-positive integer.

2: **k** – INTEGER

The function $\psi^{(k)}(x)$ to be evaluated.

Constraint: $0 \leq \mathbf{k} \leq 6$.

5.2 Optional Input Parameters

None.

5.3 Output Parameters

1: **result**

The result of the function.

2: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, **k** < 0,
or **k** > 6,
or **x** is 'too close' to a non-positive integer. That is, abs(**x** − nint(**x**)) $<$ ***machine precision*** × nint(abs(**x**)).

ifail $= 2$

The evaluation has been abandoned due to the likelihood of underflow. The result is returned as zero.

ifail $= 3$

The evaluation has been abandoned due to the likelihood of overflow. The result is returned as zero.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

All constants in nag_specfun_polygamma_deriv (s14ad) are given to approximately 18 digits of precision. If t denotes the number of digits of precision in the floating-point arithmetic being used, then clearly the maximum number in the results obtained is limited by $p = \min(t, 18)$. Empirical tests by Amos (1983) have shown that the maximum relative error is a loss of approximately two decimal places of precision. Further tests with the function $-\psi^{(0)}(x)$ have shown somewhat improved accuracy, except at points near the positive zero of $\psi^{(0)}(x)$ at $x = 1.46\ldots$, where only absolute accuracy can be obtained.

8 Further Comments

None.

9 Example

This example evaluates $\psi^{(2)}(x)$ at $x = 2.5$, and prints the results.

9.1 Program Text

```
     function s14ae_example

fprintf('s14ae example results\n\n');

x = 2.5;
k = nag_int(2);
[result, ifail] = s14ae(x, k);

disp('    x     k   (d^K/dx^K)psi(x)');
fprintf('%6.1f%5d    %12.4e\n',x,k,result);
```

9.2 Program Results

```
     s14ae example results

    x     k   (d^K/dx^K)psi(x)
   2.5    2     -2.3620e-01
```