NAG Toolbox

nag_specfun_polygamma (s14ac)

1 Purpose

nag_specfun_polygamma (s14ac) returns a value of the function $\psi(x) - \ln x$, where ψ is the psi function $\psi(x) = \dfrac{d}{dx}\ln\Gamma(x) = \dfrac{\Gamma'(x)}{\Gamma(x)}$.

2 Syntax

```
[result, ifail] = nag_specfun_polygamma(x)
[result, ifail] = s14ac(x)
```

3 Description

nag_specfun_polygamma (s14ac) returns a value of the function $\psi(x) - \ln x$. The psi function is computed without the logarithmic term so that when x is large, sums or differences of psi functions may be computed without unnecessary loss of precision, by analytically combining the logarithmic terms. For example, the difference $d = \psi\left(x + \frac{1}{2}\right) - \psi(x)$ has an asymptotic behaviour for large x given by $d \sim \ln\left(x + \frac{1}{2}\right) - \ln x + O\left(\dfrac{1}{x^2}\right) \sim \ln\left(1 + \dfrac{1}{2x}\right) \sim \dfrac{1}{2x}$.

Computing d directly would amount to subtracting two large numbers which are close to $\ln\left(x + \frac{1}{2}\right)$ and $\ln x$ to produce a small number close to $\frac{1}{2x}$, resulting in a loss of significant digits. However, using this function to compute $f(x) = \psi(x) - \ln x$, we can compute $d = f\left(x + \frac{1}{2}\right) - f(x) + \ln\left(1 + \frac{1}{2x}\right)$, and the dominant logarithmic term may be computed accurately from its power series when x is large. Thus we avoid the unnecessary loss of precision.

The function is derived from the function PSIFN in Amos (1983).

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Amos D E (1983) Algorithm 610: A portable FORTRAN subroutine for derivatives of the psi function *ACM Trans. Math. Software* **9** 494–502

5 Parameters

5.1 Compulsory Input Parameters

1: **x** – REAL (KIND=nag_wp)

The argument x of the function.

Constraint: $\mathbf{x} > 0.0$.

5.2 Optional Input Parameters

None.

5.3 Output Parameters

1: **result**

The result of the function.

2: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, **x** ≤ 0.0. nag_specfun_polygamma (s14ac) returns the value zero.

ifail $= 2$

No result is computed because underflow is likely. The value of **x** is too large. nag_specfun_polygamma (s14ac) returns the value zero.

ifail $= 3$

No result is computed because overflow is likely. The value of **x** is too small. nag_specfun_polygamma (s14ac) returns the value zero.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

All constants in nag_specfun_polygamma (s14ac) are given to approximately 18 digits of precision. Calling the number of digits of precision in the floating-point arithmetic being used t, then clearly the maximum number of correct digits in the results obtained is limited by $p = \min(t, 18)$.

With the above proviso, results returned by this function should be accurate almost to full precision, except at points close to the zero of $\psi(x)$, $x \simeq 1.461632$, where only absolute rather than relative accuracy can be obtained.

8 Further Comments

None.

9 Example

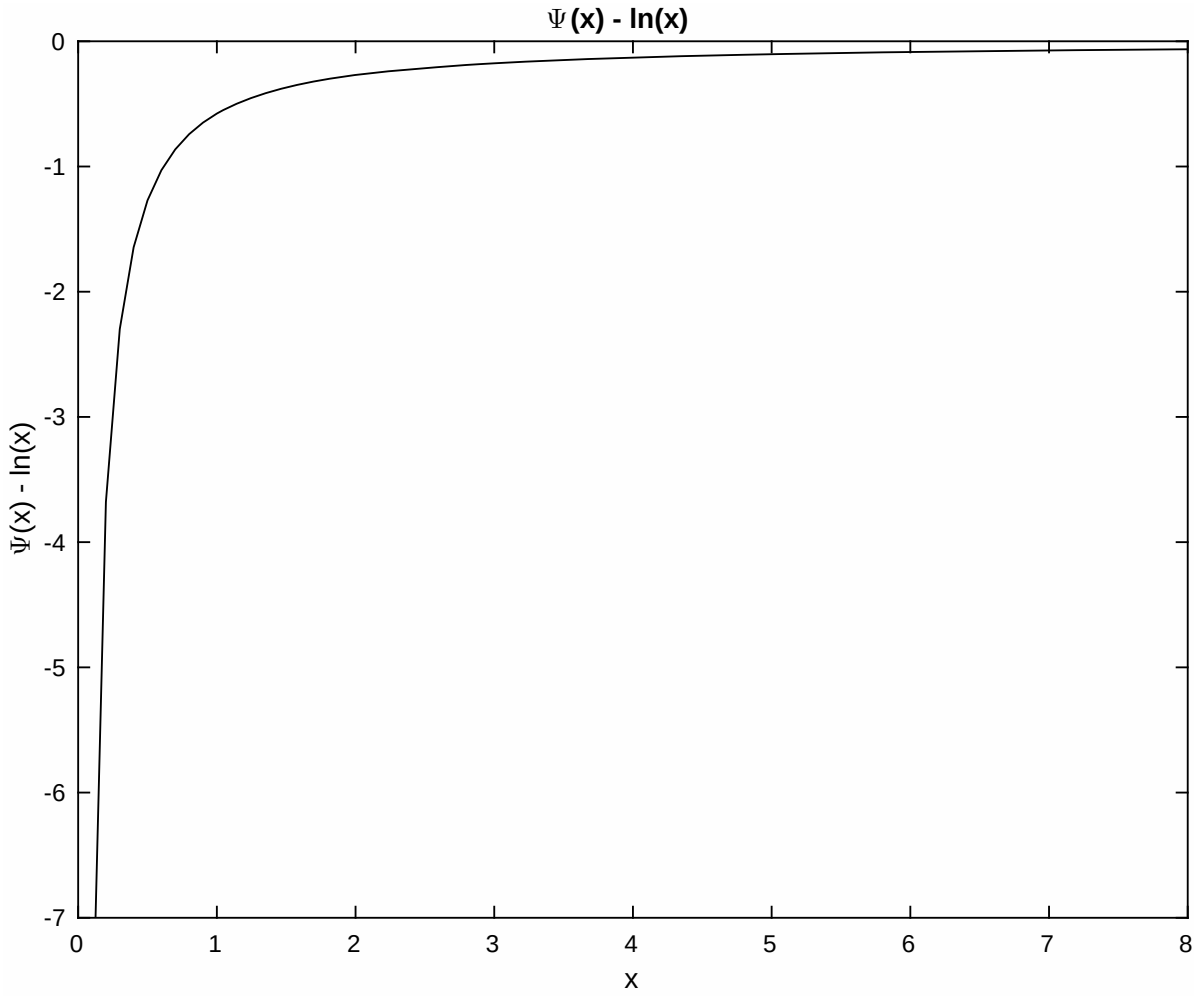
The example program reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

```
     function s14ac_example

fprintf('s14ac example results\n\n');

x = [0.1  0.5  3.6  8];
n = size(x,2);
result = x;

for j=1:n
  [result(j), ifail] = s14ac(x(j));
end

disp('        x      Psi(x)-ln(x)');
fprintf('%12.4f%12.4f\n',[x; result]);

s14ac_plot;

function s14ac_plot
  x = [0.1:0.1:8];

  for j=1:numel(x)
    [pml(j), ifail] = s14ac(x(j));
  end

  fig1 = figure;
  plot(x,pml,'-r');
  xlabel('x');
  ylabel('\Psi(x) - ln(x)');
  title('\Psi(x) - ln(x)');
  axis([0 8 -7 0]);

  % print(fig1,'-dpng','-r75','s14ac_fig1.png');
  % print(fig1,'-deps','-r75','s14ac_fig1.eps');
```

9.2 Program Results

```
     s14ac example results

        x      Psi(x)-ln(x)
      0.1000     -8.1212
      0.5000     -1.2704
      3.6000     -0.1453
      8.0000     -0.0638
```


Ψ(x) - ln(x)
Ψ(x) - ln(x)
x
0
-1
-2
-3
-4
-5
-6
-7
0
1
2
3
4
5
6
7
8