NAG Toolbox

nag_specfun_integral_cos (s13ac)

1 Purpose

nag_specfun_integral_cos (s13ac) returns the value of the cosine integral

$$\mathrm{Ci}(x) = \gamma + \ln x + \int_0^x \frac{\cos u - 1}{u} \, du, \quad x > 0$$

via the function name where γ denotes Euler's constant.

2 Syntax

```
[result, ifail] = nag_specfun_integral_cos(x)
[result, ifail] = s13ac(x)
```

3 Description

nag_specfun_integral_cos (s13ac) calculates an approximate value for $\mathrm{Ci}(x)$.

For $0 < x \leq 16$ it is based on the Chebyshev expansion

$$\mathrm{Ci}(x) = \ln x + \sum_{r=0}{}' a_r T_r(t), t = 2\left(\frac{x}{16}\right)^2 - 1.$$

For $16 < x < x_{\mathrm{hi}}$,

$$\mathrm{Ci}(x) = \frac{f(x)\sin x}{x} - \frac{g(x)\cos x}{x^2}$$

where $f(x) = \sum_{r=0} f_r T_r(t)$ and $g(x) = \sum_{r=0} g_r T_r(t)$, $t = 2\left(\frac{16}{x}\right)^2 - 1$.

For $x \geq x_{\mathrm{hi}}$, $\mathrm{Ci}(x) = 0$ to within the accuracy possible (see Section 7).

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

5.1 Compulsory Input Parameters

1: **x** – REAL (KIND=nag_wp)

The argument x of the function.

Constraint: **x** > 0.0.

5.2 Optional Input Parameters

None.

5.3 Output Parameters

1: **result**

The result of the function.

2: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

The function has been called with an argument less than or equal to zero for which the function is not defined. The result returned is zero.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

If E and ϵ are the absolute and relative errors in the result and δ is the relative error in the argument then in principle these are related by

$$|E| \simeq |\delta \cos x| \text{and } |\epsilon| \simeq \left|\frac{\delta \cos x}{\mathrm{Ci}(x)}\right|.$$

That is accuracy will be limited by ***machine precision*** near the origin and near the zeros of $\cos x$, but near the zeros of $\mathrm{Ci}(x)$ only absolute accuracy can be maintained.

The behaviour of this amplification is shown in Figure 1.

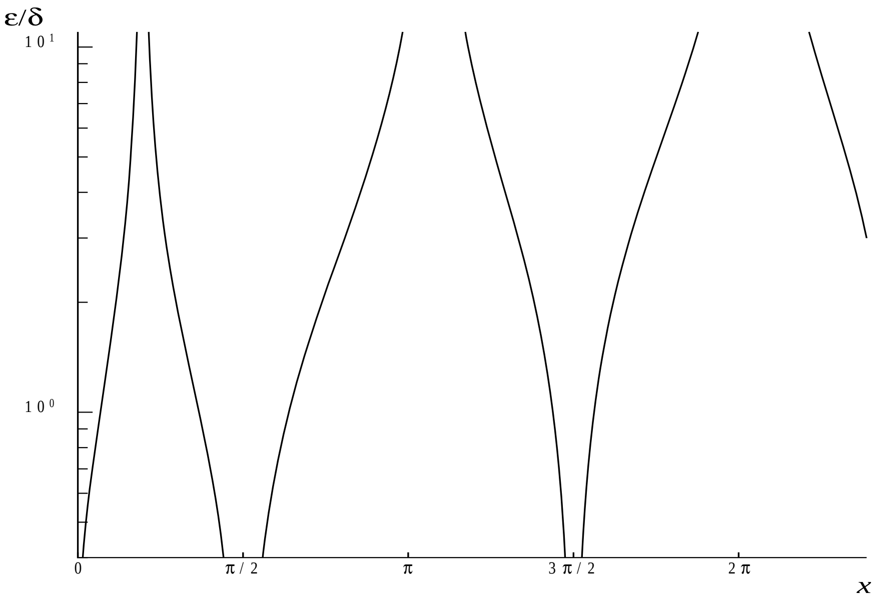

Figure 1

For large values of x, $\mathrm{Ci}(x) \sim \frac{\sin x}{x}$ therefore $\epsilon \sim \delta x \cot x$ and since δ is limited by the finite precision of the machine it becomes impossible to return results which have any relative accuracy. That is, when $x \geq 1/\delta$ we have that $|\mathrm{Ci}(x)| \leq 1/x \sim E$ and hence is not significantly different from zero.

Hence x_{hi} is chosen such that for values of $x \geq x_{\mathrm{hi}}$, $\mathrm{Ci}(x)$ in principle would have values less than the ***machine precision*** and so is essentially zero.

8 Further Comments

None.

9 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

```
function s13ac_example

fprintf('s13ac example results\n\n');

x = [0.2:0.2:1];
n = size(x,2);
result = x;

for j=1:n
  [result(j), ifail] = s13ac(x(j));
end

disp('      x           Ci(x)');
fprintf('%12.3e%12.3e\n',[x; result]);

s13ac_plot;

function s13ac_plot
  x = [0.1:0.1:2.9,3:0.5:26];
  for j=1:numel(x)
    [ci(j), ifail] = s13ac(x(j));
  end
```

```
fig1 = figure;
plot(x,ci);
xlabel('x');
ylabel('Ci(x)');
title('Cosine Integral Ci(x)');
axis([0 26 -1.5 1]);
% print(fig1,'-dpng','-r75','s13ac_fig1.png');
% print(fig1,'-deps','-r75','s13ac_fig1.eps');
```

9.2 Program Results

```
s13ac example results

   x            Ci(x)
 2.000e-01  -1.042e+00
 4.000e-01  -3.788e-01
 6.000e-01  -2.227e-02
 8.000e-01   1.983e-01
 1.000e+00   3.374e-01
```