NAG Toolbox

nag_specfun_integral_exp (s13aa)

1 Purpose

nag_specfun_integral_exp (s13aa) returns the value of the exponential integral $E_1(x)$, via the function name.

2 Syntax

```
[result, ifail] = nag_specfun_integral_exp(x)
[result, ifail] = s13aa(x)
```

3 Description

nag_specfun_integral_exp (s13aa) calculates an approximate value for

$$E_1(x) = -\mathrm{Ei}(-x) = \int_x^\infty \frac{e^{-u}}{u} du.$$

using Chebyshev expansions, where x is real. For $x < 0$, the real part of the principal value of the integral is taken. The value $E_1(0)$ is infinite, and so, when $x = 0$, nag_specfun_integral_exp (s13aa) exits with an error and returns the largest representable machine number.

For $0 < x \leq 4$,

$$E_1(x) = y(t) - \ln x = \sum_r a_r T_r(t) - \ln x,$$

where $t = \frac{1}{2}x - 1$.

For $x > 4$,

$$E_1(x) = \frac{e^{-x}}{x} y(t) = \frac{e^{-x}}{x} \sum_r a_r T_r(t),$$

where $t = -1.0 + \frac{14.5}{(x+3.25)} = \frac{11.25-x}{3.25+x}$.

In both cases, $-1 \leq t \leq +1$.

For $x < 0$, the approximation is based on expansions proposed by Cody and Thatcher Jr. (1969). Precautions are taken to maintain good relative accuracy in the vicinity of $x_0 \approx -0.372507\ldots$, which corresponds to a simple zero of $\mathrm{Ei}(-x)$.

nag_specfun_integral_exp (s13aa) guards against producing underflows and overflows by using the argument x_{hi}. To guard against overflow, if $x < -x_{\mathrm{hi}}$ the function terminates and returns the negative of the largest representable machine number. To guard against underflow, if $x > x_{\mathrm{hi}}$ the result is set directly to zero.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Cody W J and Thatcher Jr. H C (1969) Rational Chebyshev approximations for the exponential integral $\mathrm{Ei}(x)$ *Math. Comp.* **23** 289–303

5 Parameters

5.1 Compulsory Input Parameters

1: **x** – REAL (KIND=nag_wp)

The argument x of the function.

Constraint: $-x_{\mathrm{hi}} \leq \mathbf{x} < 0.0$ or $\mathbf{x} > 0.0$.

5.2 Optional Input Parameters

None.

5.3 Output Parameters

1: **result**

The result of the function.

2: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$ (*warning*)

On entry, $\mathbf{x} = 0.0$ and the function is infinite. The result returned is the largest representable machine number.

ifail $= 2$

The evaluation has been abandoned due to the likelihood of overflow. The argument $\mathbf{x} < -x_{\mathrm{hi}}$, and the result is returned as the negative of the largest representable machine number.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

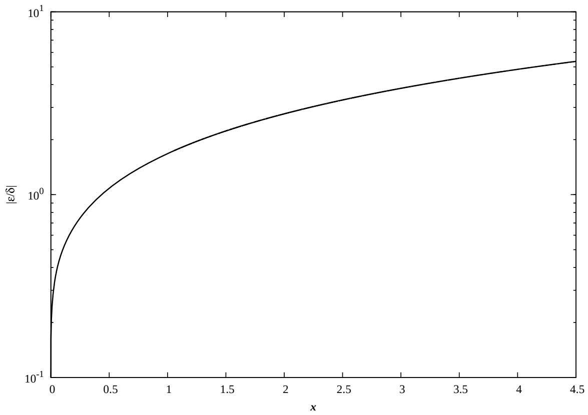
Unless stated otherwise, it is assumed that $x > 0$.

If δ and ϵ are the relative errors in argument and result respectively, then in principle,

$$|\epsilon| \simeq \left| \frac{e^{-x}}{E_1(x)} \times \delta \right|$$

so the relative error in the argument is amplified in the result by at least a factor $e^{-x}/E_1(x)$. The equality should hold if δ is greater than the ***machine precision*** (δ due to data errors etc.) but if δ is simply a result of round-off in the machine representation, it is possible that an extra figure may be lost in internal calculation and round-off.

The behaviour of this amplification factor is shown in the following graph:



It should be noted that, for absolutely small x, the amplification factor tends to zero and eventually the error in the result will be limited by ***machine precision***.

For absolutely large x,

$$\epsilon \sim x\delta = \Delta,$$

the absolute error in the argument.

For $x < 0$, empirical tests have shown that the maximum relative error is a loss of approximately 1 decimal place.

8 Further Comments

None.

9 Example

The following program reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

```
function s13aa_example

fprintf('s13aa example results\n\n');

x = [2  -1];
n = size(x,2);
result = x;

for j=1:n
  [result(j), ifail] = s13aa(x(j));
end

disp('      x          E_1(x)');
fprintf('%12.3e%12.3e\n',[x; result]);

s13aa_plot;

function s13aa_plot
  x = [-5:0.1:-0.2, -0.1:0.01:-0.01, -1e-3, -1e-4, -1e-5...
       1e-5, 1e-4, 1e-3, 0.01:0.01:0.1, 0.2:0.1:4.8];
  for j=1:numel(x)
    [e1(j), ifail] = s13aa(x(j));
  end

  fig1 = figure;
```

```
plot(x,e1);
xlabel('x');
ylabel('E_1(x)');
title('Exponential Integral E_1(x)');
% print(fig1,'-dpng','-r75','s13aa_fig1.png');
% print(fig1,'-deps','-r75','s13aa_fig1.eps');
```

9.2 Program Results

```
 s13aa example results

     x          E_1(x)
  2.000e+00   4.890e-02
 -1.000e+00  -1.895e+00
```