NAG Toolbox

nag_specfun_arcsinh (s11ab)

1 Purpose

nag_specfun_arcsinh (s11ab) returns the value of the inverse hyperbolic sine, $\operatorname{arcsinh} x$, via the function name.

2 Syntax

```
[result, ifail] = nag_specfun_arcsinh(x)
[result, ifail] = s11ab(x)
```

3 Description

nag_specfun_arcsinh (s11ab) calculates an approximate value for the inverse hyperbolic sine of its argument, $\operatorname{arcsinh} x$.

For $|x| \leq 1$ it is based on the Chebyshev expansion

$$\operatorname{arcsinh} x = x \times y(t) = x \sum_{r=0} c_r T_r(t), \qquad \text{where } t = 2x^2 - 1.$$

For $|x| > 1$ it uses the fact that

$$\operatorname{arcsinh} x = \operatorname{sign} x \times \ln\left(|x| + \sqrt{x^2+1}\right).$$

This form is used directly for $1 < |x| < 10^k$, where $k = n/2 + 1$, and the machine uses approximately n decimal place arithmetic.

For $|x| \geq 10^k$, $\sqrt{x^2+1}$ is equal to $|x|$ to within the accuracy of the machine and hence we can guard against premature overflow and, without loss of accuracy, calculate

$$\operatorname{arcsinh} x = \operatorname{sign} x \times (\ln 2 + \ln|x|).$$

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

5.1 Compulsory Input Parameters

1: **x** – REAL (KIND=nag_wp)

The argument x of the function.

5.2 Optional Input Parameters

None.

5.3 Output Parameters

1: **result**

The result of the function.

2: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

None.

7 Accuracy

If δ and ϵ are the relative errors in the argument and the result, respectively, then in principle

$$|\epsilon| \simeq \left|\frac{x}{\sqrt{1+x^2}\,\operatorname{arcsinh} x}\delta\right|.$$

That is, the relative error in the argument, x, is amplified by a factor at least $\dfrac{x}{\sqrt{1+x^2}\,\operatorname{arcsinh} x}$, in the result.

The equality should hold if δ is greater than the ***machine precision*** (δ due to data errors etc.) but if δ is simply due to round-off in the machine representation it is possible that an extra figure may be lost in internal calculation round-off.

The behaviour of the amplification factor is shown in the following graph:

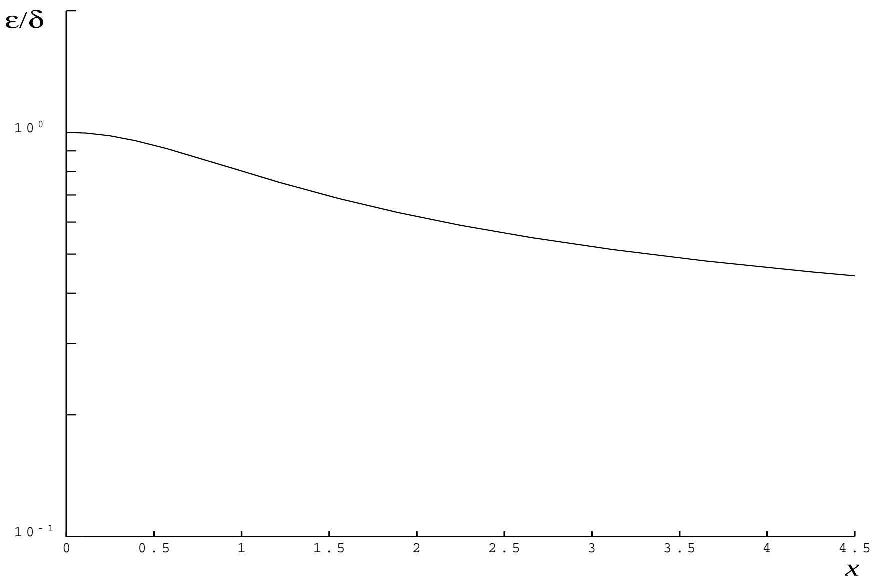

Figure 1

It should be noted that this factor is always less than or equal to one. For large x we have the absolute error in the result, E, in principle, given by

$$E \sim \delta.$$

This means that eventually accuracy is limited by ***machine precision***.

8 Further Comments

None.

9 Example

This example reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

```
function s11ab_example

fprintf('s11ab example results\n\n');

x = [-2    -0.5    1     6];
n = size(x,2);
result = x;

for j=1:n
  [result(j), ifail] = s11ab(x(j));
end

disp('      x        arcsinh(x)');
fprintf('%12.3e%12.3e\n',[x; result]);
```

9.2 Program Results

```
s11ab example results

      x        arcsinh(x)
  -2.000e+00  -1.444e+00
  -5.000e-01  -4.812e-01
   1.000e+00   8.814e-01
   6.000e+00   2.492e+00
```