

NAG Toolbox

nag_specfun_log_shifted (s01ba)

1 Purpose

nag_specfun_log_shifted (s01ba) returns a value of the shifted logarithmic function, $\ln(1+x)$, via the function name.

2 Syntax

```
[result, ifail] = nag_specfun_log_shifted(x)
[result, ifail] = s01ba(x)
```

3 Description

nag_specfun_log_shifted (s01ba) computes values of $\ln(1+x)$, retaining full relative precision even when $|x|$ is small. The function is based on the Chebyshev expansion

$$\ln \frac{1+p^2+2p\bar{x}}{1+p^2-2p\bar{x}} = 4 \sum_{k=0}^{\infty} \frac{p^{2k+1}}{2k+1} T_{2k+1}(\bar{x}).$$

Setting $\bar{x} = \frac{x(1+p^2)}{2p(x+2)}$, and choosing $p = \frac{q-1}{q+1}$, $q = \sqrt[4]{2}$ the expansion is valid in the domain $x \in \left[\frac{1}{\sqrt{2}} - 1, \sqrt{2} - 1 \right]$.

Outside this domain, $\ln(1+x)$ is computed by the standard logarithmic function.

4 References

Lyusternik L A, Chervonenkis O A and Yanpolskii A R (1965) *Handbook for Computing Elementary Functions* p. 57 Pergamon Press

5 Parameters

5.1 Compulsory Input Parameters

- 1: **x** – REAL (KIND=nag_wp)
The argument x of the function.
Constraint: $x > -1.0$.

5.2 Optional Input Parameters

None.

5.3 Output Parameters

- 1: **result**
The result of the function.
- 2: **ifail** – INTEGER
ifail = 0 unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = 1

On entry, $x \leq -1.0$.

The result is returned as zero.

ifail = -99

An unexpected error has been triggered by this routine. Please contact NAG.

ifail = -399

Your licence key may have expired or may not have been installed correctly.

ifail = -999

Dynamic memory allocation failed.

7 Accuracy

The returned result should be accurate almost to *machine precision*, with a limit of about 20 significant figures due to the precision of internal constants. Note however that if x lies very close to -1.0 and is not exact (for example if x is the result of some previous computation and has been rounded), then precision will be lost in the computation of $1 + x$, and hence $\ln(1 + x)$, in `nag_specfun_log_shifted` (s01ba).

8 Further Comments

Empirical tests show that the time taken for a call of `nag_specfun_log_shifted` (s01ba) usually lies between about 1.25 and 2.5 times the time for a call to the standard logarithm function.

9 Example

The example program reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

```
function s01ba_example

fprintf('s01ba example results\n\n');

x = [2.5 1.25e-1 -9.06e-1 1.29e-3 -7.83e-6 1.00e-9];
n = size(x,2);
result = x;

for j=1:n
    [result(j), ifail] = s01ba(x(j));
end

disp('      x      log(1+x)');
fprintf('%12.4e%12.4e\n',[x; result]);
```

9.2 Program Results

s01ba example results

x	log(1+x)
2.5000e+00	1.2528e+00
1.2500e-01	1.1778e-01
-9.0600e-01	-2.3645e+00
1.2900e-03	1.2892e-03
-7.8300e-06	-7.8300e-06
1.0000e-09	1.0000e-09
