NAG Toolbox

nag_tsa_uni_arma_roots (g13dx)

1 Purpose

nag_tsa_uni_arma_roots (g13dx) calculates the zeros of a vector autoregressive (or moving average) operator. This function is likely to be used in conjunction with nag_rand_times_mv_varma (g05pj), nag_tsa_uni_arima_resid (g13as), nag_tsa_multi_varma_estimate (g13dd) or nag_tsa_multi_varma_diag (g13ds).

2 Syntax

```
[rr, ri, rmod, ifail] = nag_tsa_uni_arma_roots(k, ip, par)
[rr, ri, rmod, ifail] = g13dx(k, ip, par)
```

3 Description

Consider the vector autoregressive moving average (VARMA) model

$$W_t - \mu = \phi_1(W_{t-1} - \mu) + \phi_2(W_{t-2} - \mu) + \cdots + \phi_p\big(W_{t-p} - \mu\big) + \epsilon_t - \theta_1\epsilon_{t-1} - \theta_2\epsilon_{t-2} - \cdots - \theta_q\epsilon_{t-q}, \quad (1)$$

where W_t denotes a vector of k time series and ϵ_t is a vector of k residual series having zero mean and a constant variance-covariance matrix. The components of ϵ_t are also assumed to be uncorrelated at non-simultaneous lags. $\phi_1, \phi_2, \ldots, \phi_p$ denotes a sequence of k by k matrices of autoregressive (AR) parameters and $\theta_1, \theta_2, \ldots, \theta_q$ denotes a sequence of k by k matrices of moving average (MA) parameters. μ is a vector of length k containing the series means. Let

$$A(\phi) = \begin{bmatrix} \phi_1 & I & 0 & . & . & . & 0 \\ \phi_2 & 0 & I & 0 & . & . & 0 \\ . & & & . & & & \\ . & & & & . & & \\ . & & & & & . & \\ \phi_{p-1} & 0 & . & . & . & 0 & I \\ \phi_p & 0 & . & . & . & 0 & 0 \end{bmatrix}_{pk \times pk}$$

where I denotes the k by k identity matrix.

The model (1) is said to be stationary if the eigenvalues of $A(\phi)$ lie inside the unit circle. Similarly let

$$B(\theta) = \begin{bmatrix} \theta_1 & I & 0 & . & . & . & 0 \\ \theta_2 & 0 & I & 0 & . & . & 0 \\ . & & & . & & & \\ . & & & & . & & \\ . & & & & & . & \\ \theta_{q-1} & 0 & . & . & . & 0 & I \\ \theta_q & 0 & . & . & . & 0 & 0 \end{bmatrix}_{qk \times qk}.$$

Then the model is said to be invertible if the eigenvalues of $B(\theta)$ lie inside the unit circle.

nag_tsa_uni_arma_roots (g13dx) returns the pk eigenvalues of $A(\phi)$ (or the qk eigenvalues of $B(\theta)$) along with their moduli, in descending order of magnitude. Thus to check for stationarity or invertibility you should check whether the modulus of the largest eigenvalue is less than one.

4 References

Wei W W S (1990) *Time Series Analysis: Univariate and Multivariate Methods* Addison–Wesley

5 Parameters

5.1 Compulsory Input Parameters

1: **k** – INTEGER

k, the dimension of the multivariate time series.

Constraint: $\mathbf{k} \geq 1$.

2: **ip** – INTEGER

The number of AR (or MA) parameter matrices, p (or q).

Constraint: $\mathbf{ip} \geq 1$.

3: **par**$(\mathbf{ip} \times \mathbf{k} \times \mathbf{k})$ – REAL (KIND=nag_wp) array

The AR (or MA) parameter matrices read in row by row in the order $\phi_1, \phi_2, \ldots, \phi_p$ (or $\theta_1, \theta_2, \ldots, \theta_q$). That is, $\mathbf{par}((l-1) \times k \times k + (i-1) \times k + j)$ must be set equal to the (i,j)th element of ϕ_l, for $l = 1, 2, \ldots, p$ (or the (i,j)th element of θ_l, for $l = 1, 2, \ldots, q$).

5.2 Optional Input Parameters

None.

5.3 Output Parameters

1: **rr**$(\mathbf{k} \times \mathbf{ip})$ – REAL (KIND=nag_wp) array

The real parts of the eigenvalues.

2: **ri**$(\mathbf{k} \times \mathbf{ip})$ – REAL (KIND=nag_wp) array

The imaginary parts of the eigenvalues.

3: **rmod**$(\mathbf{k} \times \mathbf{ip})$ – REAL (KIND=nag_wp) array

The moduli of the eigenvalues.

4: **ifail** – INTEGER

$\mathbf{ifail} = 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

$\mathbf{ifail} = 1$

On entry, $\mathbf{k} < 1$,
or $\mathbf{ip} < 1$.

$\mathbf{ifail} = 2$

An excessive number of iterations are needed to evaluate the eigenvalues of $A(\phi)$ (or $B(\theta)$). This is an unlikely exit. All output arguments are undefined.

$\mathbf{ifail} = -99$

An unexpected error has been triggered by this routine. Please contact NAG.

$\mathbf{ifail} = -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

The accuracy of the results depends on the original matrix and the multiplicity of the roots.

8 Further Comments

The time taken is approximately proportional to kp^3 (or kq^3).

9 Example

This example finds the eigenvalues of $A(\phi)$ where $k = 2$ and $p = 1$ and $\phi_1 = \begin{bmatrix} 0.802 & 0.065 \\ 0.000 & 0.575 \end{bmatrix}$.

9.1 Program Text

```
function g13dx_example

fprintf('g13dx example results\n\n');

k   = nag_int(2);
ip  = nag_int(1);
par = [0.802;     0.065;     0;     0.575];

% Calculate zeros
[rr, ri, rmod, ifail] = g13dx( ...
                              k, ip, par);

% Display results
fprintf('%20s%13s\n', 'Eigenvalues', 'Moduli');
fprintf('%20s%13s\n', '-----------', '------');
for i = 1:k*ip
  if ri(i)>=0
    fprintf(' %10.3f  + %6.3f i  %8.3f\n', rr(i), ri(i), rmod(i));
  else
    fprintf(' %10.3f  - %6.3f i  %8.3f\n', rr(i), -ri(i), rmod(i));
  end
end
```

9.2 Program Results

```
g13dx example results

        Eigenvalues       Moduli
        -----------       ------
      0.802  +  0.000 i     0.802
      0.575  +  0.000 i     0.575
```