NAG Toolbox

nag_tsa_multi_corrmat_partlag (g13dn)

1 Purpose

nag_tsa_multi_corrmat_partlag (g13dn) calculates the sample partial lag correlation matrices of a multivariate time series. A set of χ^2-statistics and their significance levels are also returned. A call to nag_tsa_multi_corrmat_cross (g13dm) is usually made prior to calling this function in order to calculate the sample cross-correlation matrices.

2 Syntax

```
[maxlag, parlag, x, pvalue, ifail] = nag_tsa_multi_corrmat_partlag(n, m, r0, r,
'k', k)
```

```
[maxlag, parlag, x, pvalue, ifail] = g13dn(n, m, r0, r, 'k', k)
```

3 Description

Let $W_t = (w_{1t}, w_{2t}, \ldots, w_{kt})^{\mathrm{T}}$, for $t = 1, 2, \ldots, n$, denote n observations of a vector of k time series. The partial lag correlation matrix at lag l, $P(l)$, is defined to be the correlation matrix between W_t and W_{t+l}, after removing the linear dependence on each of the intervening vectors $W_{t+1}, W_{t+2}, \ldots, W_{t+l-1}$. It is the correlation matrix between the residual vectors resulting from the regression of W_{t+l} on the carriers $W_{t+l-1}, \ldots, W_{t+1}$ and the regression of W_t on the same set of carriers; see Heyse and Wei (1985).

$P(l)$ has the following properties.

(i) If W_t follows a vector autoregressive model of order p, then $P(l) = 0$ for $l > p$;

(ii) When $k = 1$, $P(l)$ reduces to the univariate partial autocorrelation at lag l;

(iii) Each element of $P(l)$ is a properly normalized correlation coefficient;

(iv) When $l = 1$, $P(l)$ is equal to the cross-correlation matrix at lag 1 (a natural property which also holds for the univariate partial autocorrelation function).

Sample estimates of the partial lag correlation matrices may be obtained using the recursive algorithm described in Wei (1990). They are calculated up to lag m, which is usually taken to be at most $n/4$. Only the sample cross-correlation matrices ($\hat{R}(l)$, for $l = 0, 1, \ldots, m$) and the standard deviations of the series are required as input to nag_tsa_multi_corrmat_partlag (g13dn). These may be computed by nag_tsa_multi_corrmat_cross (g13dm). Under the hypothesis that W_t follows an autoregressive model of order $s - 1$, the elements of the sample partial lag matrix $\hat{P}(s)$, denoted by $\hat{P}_{ij}(s)$, are asymptotically Normally distributed with mean zero and variance $1/n$. In addition the statistic

$$X(s) = n\sum_{i=1}^{k}\sum_{j=1}^{k}\hat{P}_{ij}(s)^2$$

has an asymptotic χ^2-distribution with k^2 degrees of freedom. These quantities, $X(l)$, are useful as a diagnostic aid for determining whether the series follows an autoregressive model and, if so, of what order.

4 References

Heyse J F and Wei W W S (1985) The partial lag autocorrelation function *Technical Report No. 32* Department of Statistics, Temple University, Philadelphia

Wei W W S (1990) *Time Series Analysis: Univariate and Multivariate Methods* Addison–Wesley

5 Parameters

5.1 Compulsory Input Parameters

1: **n** – INTEGER

n, the number of observations in each series.

Constraint: $\mathbf{n} \geq 2$.

2: **m** – INTEGER

m, the number of partial lag correlation matrices to be computed. Note this also specifies the number of sample cross-correlation matrices that must be contained in the array **r**.

Constraint: $1 \leq \mathbf{m} < \mathbf{n}$.

3: $\mathbf{r0}(kmax, \mathbf{k})$ – REAL (KIND=nag_wp) array

$kmax$, the first dimension of the array, must satisfy the constraint $kmax \geq \mathbf{k}$.

If $i \neq j$, then $\mathbf{r0}(i,j)$ must contain the (i,j)th element of the sample cross-correlation matrix at lag zero, $\hat{R}_{ij}(0)$. If $i = j$, then $\mathbf{r0}(i,i)$ must contain the standard deviation of the ith series.

4: $\mathbf{r}(kmax, kmax, \mathbf{m})$ – REAL (KIND=nag_wp) array

$kmax$, the first dimension of the array, must satisfy the constraint $kmax \geq \mathbf{k}$.

$\mathbf{r}(i,j,l)$ must contain the (i,j)th element of the sample cross-correlation at lag l, $\hat{R}_{ij}(l)$, for $l = 1,2,\ldots,m$, $i = 1,2,\ldots,k$ and $j = 1,2,\ldots,k$, where series j leads series i (see Section 9).

5.2 Optional Input Parameters

1: **k** – INTEGER

Default: the first dimension of the arrays **r0**, **r** and the second dimension of the array **r0**. (An error is raised if these dimensions are not equal.)

k, the dimension of the multivariate time series.

Constraint: $\mathbf{k} \geq 1$.

5.3 Output Parameters

1: **maxlag** – INTEGER

The maximum lag up to which partial lag correlation matrices (along with χ^2-statistics and their significance levels) have been successfully computed. On a successful exit **maxlag** will equal **m**. If $\mathbf{ifail} = 2$ on exit, then **maxlag** will be less than **m**.

2: $\mathbf{parlag}(kmax, kmax, \mathbf{m})$ – REAL (KIND=nag_wp) array

$\mathbf{parlag}(i,j,l)$ contains the (i,j)th element of the sample partial lag correlation matrix at lag l, $\hat{P}_{ij}(l)$, for $l = 1,2,\ldots,\mathbf{maxlag}$, $i = 1,2,\ldots,k$ and $j = 1,2,\ldots,k$.

3: $\mathbf{x}(\mathbf{m})$ – REAL (KIND=nag_wp) array

$\mathbf{x}(l)$ contains the χ^2-statistic at lag l, for $l = 1,2,\ldots,\mathbf{maxlag}$.

4: $\mathbf{pvalue}(\mathbf{m})$ – REAL (KIND=nag_wp) array

$\mathbf{pvalue}(l)$ contains the significance level of the corresponding χ^2-statistic in **x**, for $l = 1,2,\ldots,\mathbf{maxlag}$.

5: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, $\mathbf{k} < 1$,
or $\mathbf{n} < 2$,
or $\mathbf{m} < 1$,
or $\mathbf{m} \geq \mathbf{n}$,
or $kmax < \mathbf{k}$,
or $lwork < (5\mathbf{m} + 6)\mathbf{k}^2 + \mathbf{k}$.

ifail $= 2$ (*warning*)

The recursive equations used to compute the sample partial lag correlation matrices have broken down at lag $\mathbf{maxlag} + 1$. All output quantities in the arrays **parlag**, **x** and **pvalue** up to and including lag **maxlag** will be correct.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

The accuracy will depend upon the accuracy of the sample cross-correlations.

8 Further Comments

The time taken is roughly proportional to m^2k^3.

If you have calculated the sample cross-correlation matrices in the arrays **r0** and **r**, without calling nag_tsa_multi_corrmat_cross (g13dm), then care must be taken to ensure they are supplied as described in Section 5. In particular, for $l \geq 1$, $\hat{R}_{ij}(l)$ must contain the sample cross-correlation coefficient between $w_{i(t-l)}$ and w_{jt}.

The function nag_tsa_multi_autocorr_part (g13db) computes squared partial autocorrelations for a specified number of lags. It may also be used to estimate a sequence of partial autoregression matrices at lags $1, 2, \ldots$ by making repeated calls to the function with the argument **nk** set to $1, 2, \ldots$. The (i, j)th element of the sample partial autoregression matrix at lag l is given by $W(i, j, l)$ when **nk** is set equal to l on entry to nag_tsa_multi_autocorr_part (g13db). Note that this is the 'Yule–Walker' estimate. Unlike the partial lag correlation matrices computed by nag_tsa_multi_corrmat_partlag (g13dn), when W_t follows an autoregressive model of order $s - 1$, the elements of the sample partial autoregressive matrix at lag s do not have variance $1/n$, making it very difficult to spot a possible cut-off point. The differences between these matrices are discussed further by Wei (1990).

Note that nag_tsa_multi_autocorr_part (g13db) takes the sample cross-covariance matrices as input whereas this function requires the sample cross-correlation matrices to be input.

9 Example

This example computes the sample partial lag correlation matrices of two time series of length 48, up to lag 10. The matrices, their χ^2-statistics and significance levels and a plot of symbols indicating which elements of the sample partial lag correlation matrices are significant are printed. Three * represent significance at the 0.5% level, two * represent significance at the 1% level and a single * represents significance at the 5% level. The * are plotted above or below the central line depending on whether the elements are significant in a positive or negative direction.

9.1 Program Text

```
     function g13dn_example

fprintf('g13dn example results\n\n');

w = [-1.49, -1.62, 5.20, 6.23, 6.21, 5.86, 4.09, 3.18, 2.62, 1.49, 1.17, ...
      0.85, -0.35, 0.24, 2.44, 2.58, 2.04, 0.40, 2.26, 3.34, 5.09, 5.00, ...
      4.78,  4.11, 3.45, 1.65, 1.29, 4.09, 6.32, 7.50, 3.89, 1.58, 5.21, ...
      5.25,  4.93, 7.38, 5.87, 5.81, 9.68, 9.07, 7.29, 7.84, 7.55, 7.32, ...
      7.97,  7.76, 7.00, 8.35;
      7.34,  6.35, 6.96, 8.54, 6.62, 4.97, 4.55, 4.81, 4.75, 4.76,10.88, ...
     10.01, 11.62,10.36, 6.40, 6.24, 7.93, 4.04, 3.73, 5.60, 5.35, 6.81, ...
      8.27,  7.68, 6.65, 6.08,10.25, 9.14,17.75,13.30, 9.63, 6.80, 4.08, ...
      5.06,  4.94, 6.65, 7.94,10.76,11.89, 5.85, 9.01, 7.50,10.02,10.38, ...
      8.15, 8.37, 10.73, 12.14];
[k,n] = size(w);
k = nag_int(k);
n = nag_int(n);
m = nag_int(10);
matrix = 'R';

% Calculate cross correlations
[wmean, r0, r, ifail] = g13dm( ...
        matrix, k, m, w);

% Calculate sample partial lag correlation matrices
[maxlag, parlag, x, pvalue, ifail] = ...
  g13dn( ...
  n, m, r0, r);

disp('Partial Lag Correlation Matrices');
for l = 1:m
  fprintf('Lag = %d\n',l);
  disp(parlag(:,:,l));
end
sn1 = 1/sqrt(double(n));
fprintf('Standard error = 1/sqrt(n) = %7.4f\n\n',sn1);

disp('Tables Of Indicator Symbols');
fprintf('\nFor Lags 1 to %d\n',m);
lhs = {'             0.005  :'; '        +     0.01   :';
       '             0.05   :';
       '   Sig. Level       :- - - - - - - - - -  Lags';
       '             0.05   :';
       '       -     0.01   :'; '             0.005  :'};
c = sn1*[3.29, 2.58, 1.96, 0, -1.96, -2.58, -3.29];
for i = 1:k
  for j=1:k
    if i==j
      fprintf('\nAuto-correlation function for series %d\n', i);
    else
      fprintf('\nCross-correlation function for series %d and series %d\n', ...
       i, j);
    end
    rhs = lhs;
    for t = 1:m
      for u = 1:3
 if parlag(i,j,t)>c(u)
   rhs{u} = strcat(rhs{u},'*');
```

```
 end
      end
      for u = 5:7
 if parlag(i,j,t)<c(u)
   rhs{u} = strcat(rhs{u},'*');
 end
      end
    end
    fprintf('\n');
    fprintf('%s\n',rhs{1:end});
  end
end

fprintf('\n Lag     Chi-square statistic     P-value\n\n');
ilag = double([1:m]);
fprintf('%4d%18.3f%19.4f\n',[ilag; x'; pvalue']);
```

9.2 Program Results

```
     g13dn example results

Partial Lag Correlation Matrices
Lag = 1
    0.7359    0.1743
    0.2114    0.5546

Lag = 2
   -0.1869   -0.0832
   -0.1805   -0.0724

Lag = 3
    0.2775   -0.0069
    0.0837   -0.2133

Lag = 4
   -0.0843    0.2269
    0.1284   -0.1764

Lag = 5
    0.2361    0.2384
   -0.0468   -0.0455

Lag = 6
   -0.0164    0.0873
    0.0996   -0.0809

Lag = 7
   -0.0355    0.2611
    0.1258    0.0120

Lag = 8
    0.0767    0.3814
    0.0268   -0.1492

Lag = 9
   -0.0651   -0.3868
    0.1887    0.0564

Lag = 10
   -0.0261   -0.2861
    0.0279   -0.1729

Standard error = 1/sqrt(n) =  0.1443

Tables Of Indicator Symbols

For Lags 1 to 10

Auto-correlation function for series 1
```

```
             0.005  :*
        +    0.01   :*
             0.05   :*
   Sig. Level       :- - - - - - - - - - -  Lags
             0.05   :
        -    0.01   :
             0.005  :

Cross-correlation function for series 1 and series 2

             0.005  :
        +    0.01   :*
             0.05   :*
   Sig. Level       :- - - - - - - - - - -  Lags
             0.05   :**
        -    0.01   :*
             0.005  :

Cross-correlation function for series 2 and series 1

             0.005  :
        +    0.01   :
             0.05   :
   Sig. Level       :- - - - - - - - - - -  Lags
             0.05   :
        -    0.01   :
             0.005  :

Auto-correlation function for series 2

             0.005  :*
        +    0.01   :*
             0.05   :*
   Sig. Level       :- - - - - - - - - - -  Lags
             0.05   :
        -    0.01   :
             0.005  :

 Lag     Chi-square statistic     P-value

   1            44.363             0.0000
   2             3.825             0.4302
   3             6.220             0.1833
   4             5.096             0.2776
   5             5.609             0.2303
   6             1.169             0.8832
   7             4.098             0.3929
   8             8.368             0.0790
   9             9.248             0.0552
  10             5.434             0.2456
```