NAG Toolbox

nag_tsa_multi_xcorr (g13bc)

1 Purpose

nag_tsa_multi_xcorr (g13bc) calculates cross-correlations between two time series.

2 Syntax

```
[s, r0, r, stat, ifail] = nag_tsa_multi_xcorr(x, y, nl, 'nxy', nxy)
[s, r0, r, stat, ifail] = g13bc(x, y, nl, 'nxy', nxy)
```

3 Description

Given two series $x_1, x_2, \ldots, x_n$ and $y_1, y_2, \ldots, y_n$ the function calculates the cross-correlations between x_t and lagged values of y_t:

$$r_{xy}(l) = \frac{\sum_{t=1}^{n-l}(x_t - \bar{x})(y_{t+l} - \bar{y})}{n s_x s_y}, \quad l = 0, 1, \ldots, L$$

where

$$\bar{x} = \frac{\sum_{t=1}^{n} x_t}{n}$$

$$s_x^2 = \frac{\sum_{t=1}^{n}(x_t - \bar{x})^2}{n}$$

and similarly for y.

The ratio of standard deviations s_y/s_x is also returned, and a portmanteau statistic is calculated:

$$\textbf{stat} = n\sum_{l=1}^{L} r_{xy}(l)^2.$$

Provided n is large, L much less than n, and both x_t, y_t are samples of series whose true autocorrelation functions are zero, then, under the null hypothesis that the true cross-correlations between the series are zero, **stat** has a χ^2-distribution with L degrees of freedom. Values of **stat** in the upper tail of this distribution provide evidence against the null hypothesis.

4 References

Box G E P and Jenkins G M (1976) *Time Series Analysis: Forecasting and Control* (Revised Edition) Holden–Day

5 Parameters

5.1 Compulsory Input Parameters

1: **x**(**nxy**) – REAL (KIND=nag_wp) array

The n values of the x series.

2: **y**(**nxy**) – REAL (KIND=nag_wp) array

The n values of the y series.

3: **nl** – INTEGER

L, the maximum lag for calculating cross-correlations.

Constraint: $1 \leq$ **nl** $<$ **nxy**.

5.2 Optional Input Parameters

1: **nxy** – INTEGER

Default: the dimension of the arrays **x**, **y**. (An error is raised if these dimensions are not equal.)

n, the length of the time series.

Constraint: **nxy** ≥ 2.

5.3 Output Parameters

1: **s** – REAL (KIND=nag_wp)

The ratio of the standard deviation of the y series to the standard deviation of the x series, s_y/s_x.

2: **r0** – REAL (KIND=nag_wp)

The cross-correlation between the x and y series at lag zero.

3: **r**(**nl**) – REAL (KIND=nag_wp) array

r(l) contains the cross-correlations between the x and y series at lags L, $r_{xy}(l)$, for $l = 1, 2, \ldots, L$.

4: **stat** – REAL (KIND=nag_wp)

The statistic for testing for absence of cross-correlation.

5: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, **nxy** ≤ 1,
or **nl** < 1,
or **nl** $\geq$ **nxy**.

ifail $= 2$

One or both of the x and y series have zero variance and hence cross-correlations cannot be calculated.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

All computations are believed to be stable.

8 Further Comments

If $n < 100$, or $L < 10\log(n)$ then the autocorrelations are calculated directly and the time taken by nag_tsa_multi_xcorr (g13bc) is approximately proportional to nL, otherwise the autocorrelations are calculated by utilizing fast Fourier transforms (FFTs) and the time taken is approximately proportional to $n\log(n)$. If FFTs are used then nag_tsa_multi_xcorr (g13bc) internally allocates approximately $6n$ real elements.

9 Example

This example reads two time series of length 20. It calculates and prints the cross-correlations up to lag 15 for the first series leading the second series and then for the second series leading the first series.

9.1 Program Text

```
function g13bc_example

fprintf('g13bc example results\n\n');

% Series
x = [0.02;  0.05;  0.08;  0.03; -0.05;  0.11; -0.01; -0.08; -0.08; -0.11;
    -0.18; -0.19; -0.09;  0.03;  0.10;  0.15; -0.14;  0.07;  0.09;  0.16];
y = [3.18;  3.21;  3.26;  3.25;  3.08;  3.01;  3.06;  3.17;  3.12;  3.04;
     3.26;  3.45;  3.33;  3.70;  3.31;  3.81;  3.33;  2.96;  3.28;  3.10];

% Number of lags
nl = nag_int(15);

% Cross correlations between x and y
[sxy, r0xy, rxy, statxy, ifail] = g13bc( ...
                                  x, y, nl);
% Cross correlations between y and x
[syx, r0yx, ryx, statyx, ifail] = g13bc( ...
                                  y, x, nl);

% Display results
fprintf('%34s%15s\n', 'Between', 'Between');
fprintf('%34s%15s\n', 'x and y', 'y and x');
fmt1 = '\n%24s%10.4f%15.4f\n';
fprintf(fmt1, 'Standard deviation ratio', sxy, syx);
fprintf('\nCross correlation at lag');
fprintf(fmt1, '0', r0xy, r0yx);
ivar = double([1:nl]');
fprintf('%24d%10.4f%15.4f\n', [ivar rxy ryx]')
fprintf(fmt1,'Test statistic          ', statxy, statyx);
```

9.2 Program Results

```
g13bc example results

                         Between        Between
                         x and y        y and x

Standard deviation ratio    2.0053         0.4987

Cross correlation at lag
                       0    0.0568         0.0568
```

1	0.0438	-0.0151
2	-0.3762	0.3955
3	-0.4864	0.3417
4	-0.6294	0.5486
5	-0.3871	0.2291
6	-0.1690	0.3190
7	-0.0678	0.1980
8	0.0962	0.0438
9	0.0788	-0.1428
10	0.2910	-0.1376
11	0.0950	-0.0387
12	0.0547	-0.0380
13	0.1855	-0.1551
14	0.0243	-0.1536
15	0.0034	-0.0696
Test statistic	22.1269	17.2917