NAG Toolbox

nag_tsa_uni_dickey_fuller_unit (g13aw)

1 Purpose

nag_tsa_uni_dickey_fuller_unit (g13aw) returns the (augmented) Dickey–Fuller unit root test.

2 Syntax

```
[ts, ifail] = nag_tsa_uni_dickey_fuller_unit(type, p, y, 'n', n)
[ts, ifail] = g13aw(type, p, y, 'n', n)
```

3 Description

If the root of the characteristic equation for a time series is one then that series is said to have a unit root. Such series are nonstationary. nag_tsa_uni_dickey_fuller_unit (g13aw) returns one of three types of (augmented) Dickey–Fuller test statistic: τ, τ_μ or τ_τ, used to test for a unit root, a unit root with drift or a unit root with drift and a deterministic time trend, respectively.

To test whether a time series, y_t, for $t = 1, 2, \ldots, n$, has a unit root the regression model

$$\nabla y_t = \beta_1 y_{t-1} + \sum_{i=1}^{p-1} \delta_i \nabla y_{t-i} + \epsilon_t$$

is fit and the test statistic τ constructed as

$$\tau = \frac{\hat{\beta}_1}{\sigma_{11}}$$

where ∇ is the difference operator, with $\nabla y_t = y_t - y_{t-1}$, and where $\hat{\beta}_1$ and σ_{11} are the least squares estimate and associated standard error for β_1 respectively.

To test for a unit root with drift the regression model

$$\nabla y_t = \beta_1 y_{t-1} + \sum_{i=1}^{p-1} \delta_i \nabla y_{t-i} + \alpha + \epsilon_t$$

is fit and the test statistic τ_μ constructed as

$$\tau_\mu = \frac{\hat{\beta}_1}{\sigma_{11}}$$

To test for a unit root with drift and deterministic time trend the regression model

$$\nabla y_t = \beta_1 y_{t-1} + \sum_{i=1}^{p-1} \delta_i \nabla y_{t-i} + \alpha + \beta_2 t + \epsilon_t$$

is fit and the test statistic τ_τ constructed as

$$\tau_\tau = \frac{\hat{\beta}_1}{\sigma_{11}}$$

The distributions of the three test statistics; τ, τ_μ and τ_τ, are nonstandard. An associated probability can be obtained from nag_stat_prob_dickey_fuller_unit (g01ew).

4 References

Dickey A D (1976) Estimation and hypothesis testing in nonstationary time series *PhD Thesis* Iowa State University, Ames, Iowa

Dickey A D and Fuller W A (1979) Distribution of the estimators for autoregressive time series with a unit root *J. Am. Stat. Assoc.* **74** **366** 427–431

5 Parameters

5.1 Compulsory Input Parameters

1: **type** – INTEGER

The type of unit test for which the probability is required.

type $= 1$
A unit root test will be performed and τ returned.

type $= 2$
A unit root test with drift will be performed and τ_μ returned.

type $= 3$
A unit root test with drift and deterministic time trend will be performed and τ_τ returned.

Constraint: **type** $= 1$, 2 or 3.

2: **p** – INTEGER

p, the degree of the autoregressive (AR) component of the Dickey–Fuller test statistic. When $p > 1$ the test is usually referred to as the augmented Dickey–Fuller test.

Constraint: $\mathbf{p} > 0$.

3: **y**(**n**) – REAL (KIND=nag_wp) array

y, the time series.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the dimension of the array **y**.

n, the length of the time series.

Constraints:

if **type** $= 1$, $\mathbf{n} > 2\mathbf{p}$;
if **type** $= 2$, $\mathbf{n} > 2\mathbf{p} + 1$;
if **type** $= 3$, $\mathbf{n} > 2\mathbf{p} + 2$.

5.3 Output Parameters

1: **ts**

2: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 11$

Constraint: **type** $= 1,\ 2$ or 3.

ifail $= 21$

Constraint: $\mathbf{p} > 0$.

ifail $= 31$

Constraint:

if **type** $= 1$, $\mathbf{n} > 2\mathbf{p}$;
if **type** $= 2$, $\mathbf{n} > 2\mathbf{p} + 1$;
if **type** $= 3$, $\mathbf{n} > 2\mathbf{p} + 2$.

ifail $= 41$

On entry, the design matrix used in the estimation of β_1 is not of full rank, this is usually due to all elements of the series being virtually identical. The returned statistic is therefore not unique and likely to be meaningless.

ifail $= 42$

$\sigma_{11} = 0$, therefore depending on the sign of $\hat{\beta}_1$, a large positive or negative value has been returned.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

None.

8 Further Comments

None.

9 Example

In this example a Dickey–Fuller unit root test is applied to a time series related to the rate of the earth's rotation about its polar axis.

9.1 Program Text

```
function g13aw_example

fprintf('g13aw example results\n\n');

% Test type
type = nag_int(1);

% Order of the AR process
p = nag_int(1);

% Time series
y = [ -217; -177; -166; -136; -110;  -95;  -64;  -37;  -14;  -25;
       -51;  -62;  -73;  -88; -113; -120;  -83;  -33;  -19;   21;
        17;   44;   44;   78;   88;  122;  126;  114;   85;   64];

% Calculate the Dickey-Fuller test statistic
[ts,ifail] = g13aw(type,p,y);

% The p-value routine can issue a warning, but still return
% sensible results, so save current warning state and turn warnings on
warn_state = nag_issue_warnings();
nag_issue_warnings(true);

% Get the associated p-value
n = nag_int(size(y,1));
[pvalue,~,ifail] = g01ew(type,n,ts);

% Reset the warning state to its initial value
nag_issue_warnings(warn_state);

% Print the results
fprintf('Dickey-Fuller test statistic     = %6.3f\n', ts);
fprintf('associated p-value               = %6.3f\n', pvalue);
```

9.2 Program Results

```
g13aw example results

Dickey-Fuller test statistic     = -2.540
associated p-value               =  0.013
```