NAG Toolbox

nag_tsa_uni_arima_forecast_state (g13ah)

1 Purpose

nag_tsa_uni_arima_forecast_state (g13ah) produces forecasts of a time series, given a time series model which has already been fitted to the time series using nag_tsa_uni_arima_estim (g13ae) or nag_tsa_uni_arima_estim_easy (g13af). The original observations are not required, since nag_tsa_uni_arima_forecast_state (g13ah) uses as input either the original state set produced by nag_tsa_uni_arima_estim (g13ae) or nag_tsa_uni_arima_estim_easy (g13af) or the state set updated by a series of new observations using nag_tsa_uni_arima_update (g13ag). Standard errors of the forecasts are also provided.

2 Syntax

```
[fva, fsd, ifail] = nag_tsa_uni_arima_forecast_state(st, mr, par, c, rms, nfv, 'nst', nst, 'npar', npar)
[fva, fsd, ifail] = g13ah(st, mr, par, c, rms, nfv, 'nst', nst, 'npar', npar)
```

3 Description

The original time series is x_t, for $t = 1, 2, \ldots, n$ and parameters have been fitted to the model of this time series using nag_tsa_uni_arima_estim (g13ae) or nag_tsa_uni_arima_estim_easy (g13af).

Forecasts of x_t, for $t = n+1, \ldots, n+L$, are calculated in five stages, as follows:

(i) set $a_t = 0$ for $t = N+1, N+2, \ldots, N+L$, where $N = n - d - (D \times s)$ is the number of differenced values in the series;

(ii) calculate the values of e_t, for $t = N+1, \ldots, N+L$, and $e_t = \phi_1 \times e_{t-1} + \cdots + \phi_p \times e_{t-p} + a_t - \theta_1 \times a_{t-1} - \cdots - \theta_q \times a_{t-q}$;

(iii) calculate the values of w_t, for $t = N+1, \ldots, N+L$, where $w_t = \Phi_1 \times w_{t-s} + \cdots + \Phi_P \times w_{t-s\times P} + e_t - \Theta_1 \times e_{t-s} - \cdots - \Theta_Q \times e_{t-s\times Q}$ and w_t for $t \leq N$ are the first $s \times P$ values in the state set, corrected for the constant;

(iv) add the constant term c to give the differenced series $\nabla^d \nabla_s^D x_t = w_t + c$, for $t = N+1, \ldots, N+L$;

(v) the differencing operations are reversed to reconstitute x_t, for $t = n+1, \ldots, n+L$.

The standard errors of these forecasts are given by $s_t = \left[V \times \left(\psi_0^2 + \psi_1^2 + \ldots + \psi_{t-n-1}^2\right)\right]^{1/2}$, for $t = n+1, \ldots, n+L$, where $\psi_0 = 1$, V is the residual variance of a_t, and ψ_j is the coefficient expressing the dependence of x_t on a_{t-j}.

To calculate ψ_j, for $j = 1, 2, \ldots, (L-1)$, the following device is used.

A copy of the state set is initialized to zero throughout and the calculations outlined above for the construction of forecasts are carried out with the settings $a_{N+1} = 1$, and $a_t = 0$, for $t = N+2, \ldots, N+L$.

The resulting quantities corresponding to the sequence $x_{N+1}, x_{N+2}, \ldots, x_{N+L}$ are precisely 1, $\psi_1, \psi_2, \ldots, \psi_{L-1}$.

The supplied time series model is used throughout these calculations, with the exception that the constant term c is taken to be zero.

4 References

None.

5 Parameters

5.1 Compulsory Input Parameters

1: **st**(**nst**) – REAL (KIND=nag_wp) array

The state set derived from nag_tsa_uni_arima_estim (g13ae) or nag_tsa_uni_arima_estim_easy (g13af) originally, or as modified using earlier calls of nag_tsa_uni_arima_update (g13ag).

2: **mr**(**7**) – INTEGER array

The orders vector (p, d, q, P, D, Q, s) of the ARIMA model, in the usual notation.

Constraints:

$p, d, q, P, D, Q, s \geq 0$;
$p + q + P + Q > 0$;
$s \neq 1$;
if $s = 0$, $P + D + Q = 0$;
if $s > 1$, $P + D + Q > 0$.

3: **par**(**npar**) – REAL (KIND=nag_wp) array

The estimates of the p values of the ϕ parameters, the q values of the θ parameters, the P values of the Φ parameters and the Q values of the Θ parameters which specify the model and which were output originally by nag_tsa_uni_arima_estim (g13ae) or nag_tsa_uni_arima_estim_easy (g13af).

4: **c** – REAL (KIND=nag_wp)

c, the value of the model constant. This will have been output by nag_tsa_uni_arima_estim (g13ae) or nag_tsa_uni_arima_estim_easy (g13af).

5: **rms** – REAL (KIND=nag_wp)

V, the residual variance associated with the model.

If nag_tsa_uni_arima_estim_easy (g13af) was used to estimate the model, **rms** should be set to **s**/**ndf**, where **s** and **ndf** were output by nag_tsa_uni_arima_estim_easy (g13af).

If nag_tsa_uni_arima_estim (g13ae) was used to estimate the model, **rms** should be set to **s**/**icount**(5), where **s** and **icount**(5) were output by nag_tsa_uni_arima_estim (g13ae).

Constraint: **rms** ≥ 0.0.

6: **nfv** – INTEGER

L, the required number of forecasts.

Constraint: **nfv** > 0.

5.2 Optional Input Parameters

1: **nst** – INTEGER

Default: the dimension of the array **st**.

The number of values in the state set array **st**.

Constraint: **nst** $= P \times s + D \times s + d + q + \max(p, Q \times s)$. (As returned by nag_tsa_uni_arima_estim (g13ae) or nag_tsa_uni_arima_estim_easy (g13af)).

2: **npar** – INTEGER

Default: the dimension of the array **par**.

The number of ϕ, θ, Φ and Θ parameters in the model.

Constraint: **npar** $= p + q + P + Q$.

5.3 Output Parameters

1: **fva**(**nfv**) – REAL (KIND=nag_wp) array

nfv forecast values relating to the original undifferenced series.

2: **fsd**(**nfv**) – REAL (KIND=nag_wp) array

The standard errors associated with each of the **nfv** forecast values in **fva**.

3: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, **npar** $\neq p + q + P + Q$,
or the orders vector **mr** is invalid (check the constraints given in Section 5).

ifail $= 2$

On entry, **nst** $\neq P \times s + D \times s + d + q + \max(Q \times s, p)$.

ifail $= 3$

On entry, **nfv** ≤ 0.

ifail $= 4$

On entry, $nwa < 4 \times$ **npar** $+ 3 \times$ **nst**.

ifail $= 5$

On entry, **rms** < 0.0.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

The computations are believed to be stable.

8 Further Comments

The time taken by nag_tsa_uni_arima_forecast_state (g13ah) is approximately proportional to **nfv** $\times$ **npar**.

9 Example

The following program is based on the data derived in the example used to illustrate nag_tsa_uni_arima_update (g13ag).

These consist of a set of orders indicating that there are two moving average parameters (one non-seasonal, and one seasonal with periodicity 12).

The model constant is zero.

The state set contains 26 values.

In addition the residual mean-square derived when the model was originally fitted is given.

Twelve forecasts and their associated errors are obtained.

9.1 Program Text

```
function g13ah_example

fprintf('g13ah example results\n\n');

% state set from g13ae, g13af or g13ag
st = [0.0660;    -0.0513;     0.1715;    -0.0249;     0.0588;
      0.1167;     0.1493;     0.0199;    -0.1884;    -0.1289;
     -0.1172;     0.1122;     6.0039;     0.0443;    -0.0070;
      0.0252;     0.0020;     0.0353;    -0.0460;     0.0374;
      0.0151;    -0.0237;     0.0031;     0.0188;     0.0066;
      0.0125];

% Orders
mr = [nag_int(0);1;1;0;1;1;12];

% Parameter estimates
par = [0.327; 0.6262];

% residual variance
rms = 0.0014;

c = 0;
nfv = nag_int(12);

% Produce forecasts
[fva, fsd, ifail] = g13ah( ...
                          st, mr, par, c, rms, nfv);

fprintf('The required %4d forecast values are as follows\n', nfv);
for j = 1:8:nfv
  fprintf('%8.4f', fva(j:min(j+7,nfv)));
  fprintf('\n');
end
fprintf('\nThe standard deviations corresponding to the forecasts are\n');
for j = 1:8:nfv
  fprintf('%8.4f', fsd(j:min(j+7,nfv)));
  fprintf('\n');
end
```

9.2 Program Results

```
g13ah example results

The required   12 forecast values are as follows
  6.0381  5.9912  6.1469  6.1207  6.1574  6.3029  6.4288  6.4392
  6.2657  6.1348  6.0059  6.1139

The standard deviations corresponding to the forecasts are
  0.0374  0.0451  0.0517  0.0575  0.0627  0.0676  0.0721  0.0764
  0.0805  0.0843  0.0880  0.0915
```