NAG Toolbox

nag_rand_multivar_students_t (g05ry)

1 Purpose

nag_rand_multivar_students_t (g05ry) sets up a reference vector and generates an array of pseudorandom numbers from a multivariate Student's t distribution with ν degrees of freedom, mean vector a and covariance matrix $\frac{\nu}{\nu-2}C$.

2 Syntax

```
[r, state, x, ifail] = nag_rand_multivar_students_t(mode, n, df, xmu, c, r, state, 'm', m, 'lr', lr)
[r, state, x, ifail] = g05ry(mode, n, df, xmu, c, r, state, 'm', m, 'lr', lr)
```

3 Description

When the covariance matrix is nonsingular (i.e., strictly positive definite), the distribution has probability density function

$$f(x) = \frac{\Gamma\left(\frac{(\nu+m)}{2}\right)}{(\pi v)^{m/2}\Gamma(\nu/2)|C|^{\frac{1}{2}}}\left[1 + \frac{(x-a)^{\mathrm{T}}C^{-1}(x-a)}{\nu}\right]^{\frac{-(\nu+m)}{2}}$$

where m is the number of dimensions, ν is the degrees of freedom, a is the vector of means, x is the vector of positions and $\frac{\nu}{\nu-2}C$ is the covariance matrix.

The function returns the value

$$x = a + \sqrt{\frac{\nu}{s}}z$$

where z is generated by nag_rand_dist_normal (g05sk) from a Normal distribution with mean zero and covariance matrix C and s is generated by nag_rand_dist_chisq (g05sd) from a χ^2-distribution with ν degrees of freedom.

One of the initialization functions nag_rand_init_repeat (g05kf) (for a repeatable sequence if computed sequentially) or nag_rand_init_nonrepeat (g05kg) (for a non-repeatable sequence) must be called prior to the first call to nag_rand_multivar_students_t (g05ry).

4 References

Knuth D E (1981) *The Art of Computer Programming (Volume 2)* (2nd Edition) Addison–Wesley

Wilkinson J H (1965) *The Algebraic Eigenvalue Problem* Oxford University Press, Oxford

5 Parameters

5.1 Compulsory Input Parameters

1: **mode** – INTEGER

A code for selecting the operation to be performed by the function.

mode $= 0$
Set up reference vector only.

mode $= 1$
Generate variates using reference vector set up in a prior call to nag_rand_multivar_students_t (g05ry).

mode $= 2$
Set up reference vector and generate variates.

Constraint: **mode** $= 0$, 1 or 2.

2: **n** – INTEGER

n, the number of random variates required.

Constraint: **n** ≥ 0.

3: **df** – INTEGER

ν, the number of degrees of freedom of the distribution.

Constraint: **df** ≥ 3.

4: **xmu**(**m**) – REAL (KIND=nag_wp) array

a, the vector of means of the distribution.

5: **c**(ldc, **m**) – REAL (KIND=nag_wp) array

ldc, the first dimension of the array, must satisfy the constraint $ldc \geq$ **m**.

Matrix which, along with **df**, defines the covariance of the distribution. Only the upper triangle need be set.

Constraint: **c** must be positive semidefinite to ***machine precision***.

6: **r**(**lr**) – REAL (KIND=nag_wp) array

If **mode** $= 1$, the reference vector as set up by nag_rand_multivar_students_t (g05ry) in a previous call with **mode** $= 0$ or 2.

7: **state**(:) – INTEGER array

Note: the actual argument supplied **must** be the array **state** supplied to the initialization routines nag_rand_init_repeat (g05kf) or nag_rand_init_nonrepeat (g05kg).

Contains information on the selected base generator and its current state.

5.2 Optional Input Parameters

1: **m** – INTEGER

Default: the dimension of the array **xmu** and the first dimension of the array **c** and the second dimension of the array **c**. (An error is raised if these dimensions are not equal.)

m, the number of dimensions of the distribution.

Constraint: **m** > 0.

2: **lr** – INTEGER

Default: the dimension of the array **r**.

The dimension of the array **r**. if **mode** $= 1$, it must be the same as the value of **lr** specified in the prior call to nag_rand_multivar_students_t (g05ry) with **mode** $= 0$ or 2.

Constraint: **lr** $\geq$ **m** $\times$ (**m** $+ 1$) $+ 2$.

5.3 Output Parameters

1: **r**(**lr**) – REAL (KIND=nag_wp) array

If **mode** $= 0$ or 2, the reference vector that can be used in subsequent calls to nag_rand_multivar_students_t (g05ry) with **mode** $= 1$.

2: **state**(:) – INTEGER array

Contains updated information on the state of the generator.

3: $\mathbf{x}(ldx, \mathbf{m})$ – REAL (KIND=nag_wp) array

The array of pseudorandom multivariate Student's t vectors generated by the function, with $\mathbf{x}(i, j)$ holding the jth dimension for the ith variate.

4: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

Constraint: **mode** $= 0$, 1 or 2.

ifail $= 2$

Constraint: **n** ≥ 0.

ifail $= 3$

Constraint: **df** ≥ 3.

ifail $= 4$

Constraint: **m** > 0.

ifail $= 6$

On entry, the covariance matrix C is not positive semidefinite to ***machine precision***.

ifail $= 7$

Constraint: $ldc \geq$ **m**.

ifail $= 8$

m is not the same as when **r** was set up in a previous call.

ifail $= 9$

On entry, **lr** is not large enough, **lr** $= \langle value \rangle$: minimum length required .

ifail $= 10$

On entry, **state** vector has been corrupted or not initialized.

ifail $= 12$

Constraint: $ldx \geq$ **n**.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

Not applicable.

8 Further Comments

The time taken by nag_rand_multivar_students_t (g05ry) is of order nm^3.

It is recommended that the diagonal elements of C should not differ too widely in order of magnitude. This may be achieved by scaling the variables if necessary. The actual matrix decomposed is $C + E = LL^{\mathrm{T}}$, where E is a diagonal matrix with small positive diagonal elements. This ensures that, even when C is singular, or nearly singular, the Cholesky factor L corresponds to a positive definite covariance matrix that agrees with C within ***machine precision***.

9 Example

This example prints ten pseudorandom observations from a multivariate Student's t-distribution with ten degrees of freedom, means vector

$$\begin{bmatrix} 1.0 \\ 2.0 \\ -3.0 \\ 0.0 \end{bmatrix}$$

and **c** matrix

$$\begin{bmatrix} 1.69 & 0.39 & -1.86 & 0.07 \\ 0.39 & 98.01 & -7.07 & -0.71 \\ -1.86 & -7.07 & 11.56 & 0.03 \\ 0.07 & -0.71 & 0.03 & 0.01 \end{bmatrix},$$

generated by nag_rand_multivar_students_t (g05ry). All ten observations are generated by a single call to nag_rand_multivar_students_t (g05ry) with **mode** $= 2$. The random number generator is initialized by nag_rand_init_repeat (g05kf).

9.1 Program Text

```
function g05ry_example

fprintf('g05ry example results\n\n');

% Initialize the base generator to a repeatable sequence
seed  = [nag_int(1762543)];
genid = nag_int(1);
subid = nag_int(1);
[state, ifail] = g05kf( ...
                        genid, subid, seed);

% Number of variates and degrees of freedom
n  = nag_int(10);
df = nag_int(10);
```

```
% Distribution means
xmu = [1; 2; -3; 0];

% Upper triangular part of covariance matrix
c = [ 1.69,  0.39, -1.86,  0.07;
      0,    98.01, -7.07, -0.71;
      0,     0,    11.56,  0.03;
      0,     0,     0,     0.01];
m = size(c,1);

% Setup and generate in one go
mode = nag_int(2);

% Generate variates from a multivariate Student t distrobution
lr = m*(m+1) + 2;
r = zeros(lr, 1);
[r, state, x, ifail] = g05ry( ...
                              mode, n, df, xmu, c, r, state);

disp('Variates from multivariate Student t distribution');
disp(x);
```

9.2 Program Results

```
     g05ry example results

Variates from multivariate Student t distribution
    1.4957  -15.6226   -3.8101    0.1294
   -1.0827   -6.7473    0.6696   -0.0391
    2.1369    6.3861   -5.7413    0.0140
    2.2481  -16.0417   -1.0982    0.1641
   -0.2550    3.5166   -0.2541   -0.0592
    0.9731   -4.3553   -4.4181    0.0043
    0.7098   -3.4281    1.1741    0.0586
    1.8827   23.2619    1.5140   -0.0704
    0.9904   22.7479    0.1811   -0.0893
    1.5026    2.7753   -2.2805   -0.0112
```