

NAG Toolbox

nag_rand_copula_frank (g05rj)

1 Purpose

nag_rand_copula_frank (g05rj) generates pseudorandom uniform variates with joint distribution of a Frank Archimedean copula.

2 Syntax

```
[state, x, ifail] = nag_rand_copula_frank(n, m, theta, sorder, state)
[state, x, ifail] = g05rj(n, m, theta, sorder, state)
```

3 Description

Generates n pseudorandom uniform m -variates whose joint distribution is the Frank Archimedean copula C_θ , given by

$$C_\theta = -\frac{1}{\theta} \ln \left[1 + \frac{(e^{-\theta u_1} - 1)(e^{-\theta u_2} - 1) \cdots (e^{-\theta u_m} - 1)}{(e^{-\theta} - 1)^{m-1}} \right], \quad \begin{cases} \theta \in (0, \infty), \\ u_j \in (0, 1], \quad j = 1, \dots, m; \end{cases}$$

with the special case:

$$C_\infty = \min(u_1, u_2, \dots, u_m), \text{ the Fréchet–Hoeffding upper bound.}$$

The generation method uses mixture of powers.

One of the initialization functions nag_rand_init_repeat (g05kf) (for a repeatable sequence if computed sequentially) or nag_rand_init_nonrepeat (g05kg) (for a non-repeatable sequence) must be called prior to the first call to nag_rand_copula_frank (g05rj).

4 References

Marshall A W and Olkin I (1988) Families of multivariate distributions *Journal of the American Statistical Association* **83** 403

Nelsen R B (2006) *An Introduction to Copulas* (2nd Edition) Springer Series in Statistics

5 Parameters

5.1 Compulsory Input Parameters

1: **n** – INTEGER

n , the number of pseudorandom uniform variates to generate.

Constraint: **n** ≥ 0 .

2: **m** – INTEGER

m , the number of dimensions.

Constraint: **m** ≥ 2 .

3: **theta** – REAL (KIND=nag_wp)

θ , the copula parameter.

Constraint: **theta** $\geq 1.0 \times 10^{-6}$.

4: **sorder** – INTEGER

Determines the storage order of variates; the (i, j) th variate is stored in $\mathbf{x}(i, j)$ if **sorder** = 1, and $\mathbf{x}(j, i)$ if **sorder** = 2, for $i = 1, 2, \dots, n$ and $j = 1, 2, \dots, m$.

Constraint: **sorder** = 1 or 2.

5: **state**(:) – INTEGER array

Note: the actual argument supplied **must** be the array **state** supplied to the initialization routines `nag_rand_init_repeat` (g05kf) or `nag_rand_init_nonrepeat` (g05kg).

Contains information on the selected base generator and its current state.

5.2 Optional Input Parameters

None.

5.3 Output Parameters

1: **state**(:) – INTEGER array

Contains updated information on the state of the generator.

2: $\mathbf{x}(ldx, sdx)$ – REAL (KIND=nag_wp) array

The pseudorandom uniform variates with joint distribution described by C_θ , with $\mathbf{x}(i, j)$ holding the i th value for the j th dimension if **sorder** = 1 and the j th value for the i th dimension of **sorder** = 2.

3: **ifail** – INTEGER

ifail = 0 unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = 1

On entry, corrupt **state** argument.

ifail = 2

Constraint: **theta** $\geq 1.0 \times 10^{-6}$.

ifail = 3

Constraint: **n** ≥ 0 .

ifail = 4

Constraint: **m** ≥ 2 .

ifail = 5

On entry, invalid **sorder**.
Constraint: **sorder** = 1 or 2.

ifail = 7

On entry, ldx is too small: $ldx = \langle value \rangle$.

ifail = 8

On entry, *sdx* is too small: *sdx* = $\langle value \rangle$.

ifail = -99

An unexpected error has been triggered by this routine. Please contact NAG.

ifail = -399

Your licence key may have expired or may not have been installed correctly.

ifail = -999

Dynamic memory allocation failed.

7 Accuracy

Not applicable.

8 Further Comments

In practice, the need for numerical stability restricts the range of θ such that:

the function requires $\theta \geq 1.0 \times 10^{-6}$;

if $\theta > -\ln \epsilon$, the function returns pseudorandom uniform variates with C_∞ joint distribution;

where ϵ is the *machine precision* returned by `nag_machine_precision` (x02aj).

9 Example

This example generates thirteen four-dimensional variates for copula $C_{4.0}$.

9.1 Program Text

```
function g05rj_example

fprintf('g05rj example results\n\n');

% Initialize the base generator to a repeatable sequence
seed = [nag_int(1762543)];
genid = nag_int(1);
subid = nag_int(1);
[state, ifail] = g05kf( ...
                    genid, subid, seed);

% Sample size
n = nag_int(13);
m = nag_int(4);
% Sample order
sorder = nag_int(1);

% Parameter
theta = 4;

% Generate variates
[state, x, ifail] = g05rj( ...
                        n, m, theta, sorder, state);

disp('Variates from a Frank copula');
disp(x);
```

9.2 Program Results

g05rj example results

Variates from a Frank copula

0.5679	0.1977	0.8682	0.2664
0.0965	0.3532	0.9773	0.3102
0.5526	0.2562	0.6341	0.6267
0.8036	0.4747	0.7310	0.5515
0.2043	0.9797	0.3628	0.4968
0.4777	0.8146	0.3922	0.4005
0.4162	0.5002	0.5074	0.2008
0.3703	0.0971	0.0527	0.0278
0.4354	0.4880	0.4096	0.4259
0.2693	0.1169	0.0639	0.1555
0.0127	0.3080	0.2352	0.4659
0.0730	0.3239	0.2020	0.0568
0.2369	0.0817	0.3118	0.4370
