NAG Toolbox

nag_rand_copula_clayton_bivar (g05re)

1 Purpose

nag_rand_copula_clayton_bivar (g05re) generates pseudorandom uniform bivariates with joint distribution of a Clayton/Cook–Johnson Archimedean copula.

2 Syntax

```
[state, x, ifail] = nag_rand_copula_clayton_bivar(n, theta, sorder, state)
[state, x, ifail] = g05re(n, theta, sorder, state)
```

3 Description

Generates pseudorandom uniform bivariates $\{u_1, u_2\} \in (0,1]^2$ whose joint distribution is the Clayton/Cook–Johnson Archimedean copula C_θ with parameter θ, given by

$$C_\theta = \left[\max\left({u_1}^{-\theta} + {u_2}^{-\theta} - 1, 0\right)\right]^{-1/\theta}, \quad \theta \in (-1, \infty) \setminus \{0\}$$

with the special cases:

$C_{-1} = \max(u_1 + u_2 - 1, 0)$, the Fréchet–Hoeffding lower bound;

$C_0 = u_1 u_2$, the product copula;

$C_\infty = \min(u_1, u_2)$, the Fréchet–Hoeffding upper bound.

The generation method uses conditional sampling.

One of the initialization functions nag_rand_init_repeat (g05kf) (for a repeatable sequence if computed sequentially) or nag_rand_init_nonrepeat (g05kg) (for a non-repeatable sequence) must be called prior to the first call to nag_rand_copula_clayton_bivar (g05re).

4 References

Nelsen R B (2006) *An Introduction to Copulas* (2nd Edition) Springer Series in Statistics

5 Parameters

5.1 Compulsory Input Parameters

1: **n** – INTEGER

n, the number of bivariates to generate.

Constraint: $\mathbf{n} \geq 0$.

2: **theta** – REAL (KIND=nag_wp)

θ, the copula parameter.

Constraint: $\mathbf{theta} \geq -1.0$.

3: **sorder** – INTEGER

Determines the storage order of variates; the (i,j)th variate is stored in $\mathbf{x}(i,j)$ if $\mathbf{sorder} = 1$, and $\mathbf{x}(j,i)$ if $\mathbf{sorder} = 2$, for $i = 1, 2, \ldots, n$ and $j = 1, 2$.

Constraint: $\mathbf{sorder} = 1$ or 2.

4: **state**(:) – INTEGER array

Note: the actual argument supplied **must** be the array **state** supplied to the initialization routines nag_rand_init_repeat (g05kf) or nag_rand_init_nonrepeat (g05kg).

Contains information on the selected base generator and its current state.

5.2 Optional Input Parameters

None.

5.3 Output Parameters

1: **state**(:) – INTEGER array

Contains updated information on the state of the generator.

2: $\mathbf{x}(ldx, sdx)$ – REAL (KIND=nag_wp) array

The n bivariate uniforms with joint distribution described by C_θ, with $\mathbf{x}(i, j)$ holding the ith value for the jth dimension if **sorder** $= 1$ and the jth value for the ith dimension if **sorder** $= 2$.

3: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, corrupt **state** argument.

ifail $= 2$

Constraint: **theta** ≥ -1.0.

ifail $= 3$

Constraint: **n** ≥ 0.

ifail $= 4$

On entry, invalid **sorder**.
Constraint: **sorder** $= 1$ or 2.

ifail $= 6$

On entry, ldx is too small: $ldx = \langle value \rangle$.

ifail $= 7$

On entry, sdx is too small: $sdx = \langle value \rangle$.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

Not applicable.

8 Further Comments

In practice, the need for numerical stability restricts the range of θ such that:

if $(\theta + 1) < \epsilon$, the function returns pseudorandom uniform variates with C_{-1} joint distribution;

if $|\theta| < 1.0 \times 10^{-6}$, the function returns pseudorandom uniform variates with C_0 joint distribution;

if $\theta > \ln \epsilon_s / \ln\left(1.0 \times 10^{-2}\right)$, the function returns pseudorandom uniform variates with C_∞ joint distribution;

where ϵ_s is the safe-range parameter, the value of which is returned by nag_machine_real_safe (x02am); and ϵ is the ***machine precision*** returned by nag_machine_precision (x02aj).

9 Example

This example generates thirteen variates for copula $C_{-0.8}$.

9.1 Program Text

```
function g05re_example

fprintf('g05re example results\n\n');

% Initialize the base generator to a repeatable sequence
seed  = [nag_int(1762543)];
genid = nag_int(1);
subid = nag_int(1);
[state, ifail] = g05kf( ...
                        genid, subid, seed);

% Sample size and order
n      = nag_int(13);
sorder = nag_int(1);

% Parameter
theta = -0.8;

% Generate variates
[state, x, ifail] = g05re( ...
                           n, theta, sorder, state);

disp('Variates from bivariate Clayton/Cook--Johnson copula');
disp(x);
```

9.2 Program Results

```
g05re example results

Variates from bivariate Clayton/Cook--Johnson copula
    0.6400    0.2223
    0.1154    0.8101
    0.7486    0.1439
    0.8003    0.1062
    0.1135    0.9946
    0.4975    0.7655
    0.3904    0.4925
```

```
    0.7892    0.1196
    0.5032    0.4116
    0.6750    0.2093
    0.0600    0.9055
    0.2655    0.7085
    0.6276    0.2370
```