NAG Toolbox

nag_rand_copula_normal (g05rd)

1 Purpose

nag_rand_copula_normal (g05rd) sets up a reference vector and generates an array of pseudorandom numbers from a Normal (Gaussian) copula with covariance matrix C.

2 Syntax

```
[r, state, x, ifail] = nag_rand_copula_normal(mode, n, c, r, state, 'm', m, 'lr', lr)
[r, state, x, ifail] = g05rd(mode, n, c, r, state, 'm', m, 'lr', lr)
```

3 Description

The Gaussian copula, G, is defined by

$$G(u_1, u_2, \ldots, u_m; C) = \Phi_C\left(\phi^{-1}_{C_{11}}(u_1), \phi^{-1}_{C_{22}}(u_2), \ldots, \phi^{-1}_{C_{mm}}(u_m)\right)$$

where m is the number of dimensions, Φ_C is the multivariate Normal density function with mean zero and covariance matrix C and $\phi^{-1}_{C_{ii}}$ is the inverse of the univariate Normal density function with mean zero and variance C_{ii}.

nag_rand_multivar_normal (g05rz) is used to generate a vector from a multivariate Normal distribution and nag_stat_prob_normal (g01ea) is used to convert each element of that vector into a uniformly distributed value between zero and one.

One of the initialization functions nag_rand_init_repeat (g05kf) (for a repeatable sequence if computed sequentially) or nag_rand_init_nonrepeat (g05kg) (for a non-repeatable sequence) must be called prior to the first call to nag_rand_copula_normal (g05rd).

4 References

Nelsen R B (1998) *An Introduction to Copulas. Lecture Notes in Statistics 139* Springer

Sklar A (1973) Random variables: joint distribution functions and copulas *Kybernetika* **9** 499–460

5 Parameters

5.1 Compulsory Input Parameters

1: **mode** – INTEGER

A code for selecting the operation to be performed by the function.

mode $= 0$
Set up reference vector only.

mode $= 1$
Generate variates using reference vector set up in a prior call to nag_rand_copula_normal (g05rd).

mode $= 2$
Set up reference vector and generate variates.

Constraint: **mode** $= 0$, 1 or 2.

2: **n** – INTEGER

n, the number of random variates required.

Constraint: $\mathbf{n} \geq 0$.

3: **c**(ldc, **m**) – REAL (KIND=nag_wp) array

ldc, the first dimension of the array, must satisfy the constraint $ldc \geq \mathbf{m}$.

The covariance matrix of the distribution. Only the upper triangle need be set.

Constraint: C must be positive semidefinite to ***machine precision***.

4: **r**(**lr**) – REAL (KIND=nag_wp) array

If **mode** $= 1$, the reference vector as set up by nag_rand_copula_normal (g05rd) in a previous call with **mode** $= 0$ or 2.

5: **state**(:) – INTEGER array

Note: the actual argument supplied **must** be the array **state** supplied to the initialization routines nag_rand_init_repeat (g05kf) or nag_rand_init_nonrepeat (g05kg).

Contains information on the selected base generator and its current state.

5.2 Optional Input Parameters

1: **m** – INTEGER

Default: the first dimension of the array **c** and the second dimension of the array **c**. (An error is raised if these dimensions are not equal.)

m, the number of dimensions of the distribution.

Constraint: $\mathbf{m} > 0$.

2: **lr** – INTEGER

Default: the dimension of the array **r**.

The dimension of the array **r**. if **mode** $= 1$, it must be the same as the value of **lr** specified in the prior call to nag_rand_copula_normal (g05rd) with **mode** $= 0$ or 2.

Constraint: $\mathbf{lr} \geq \mathbf{m} \times (\mathbf{m} + 1) + 1$.

5.3 Output Parameters

1: **r**(**lr**) – REAL (KIND=nag_wp) array

If **mode** $= 0$ or 2, the reference vector that can be used in subsequent calls to nag_rand_copula_normal (g05rd) with **mode** $= 1$.

2: **state**(:) – INTEGER array

Contains updated information on the state of the generator.

3: **x**(ldx, **m**) – REAL (KIND=nag_wp) array

The array of values from a multivariate Gaussian copula, with $\mathbf{x}(i, j)$ holding the jth dimension for the ith variate.

4: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

Constraint: **mode** $= 0$, 1 or 2.

ifail $= 2$

Constraint: **n** ≥ 0.

ifail $= 3$

Constraint: **m** > 0.

ifail $= 4$

On entry, the covariance matrix C is not positive semidefinite to ***machine precision***.

ifail $= 5$

Constraint: $ldc \geq$ **m**.

ifail $= 6$

m is not the same as when **r** was set up in a previous call.

ifail $= 7$

On entry, **lr** is not large enough, **lr** $= \langle value \rangle$: minimum length required .

ifail $= 8$

On entry, **state** vector has been corrupted or not initialized.

ifail $= 10$

Constraint: $ldx \geq$ **n**.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

See Section 7 in nag_rand_multivar_normal (g05rz) for an indication of the accuracy of the underlying multivariate Normal distribution.

8 Further Comments

The time taken by nag_rand_copula_normal (g05rd) is of order nm^3.

It is recommended that the diagonal elements of C should not differ too widely in order of magnitude. This may be achieved by scaling the variables if necessary. The actual matrix decomposed is $C + E = LL^{\mathrm{T}}$, where E is a diagonal matrix with small positive diagonal elements. This ensures that,

even when C is singular, or nearly singular, the Cholesky factor L corresponds to a positive definite covariance matrix that agrees with C within ***machine precision***.

9 Example

This example prints ten pseudorandom observations from a Normal copula with covariance matrix

$$\begin{bmatrix} 1.69 & 0.39 & -1.86 & 0.07 \\ 0.39 & 98.01 & -7.07 & -0.71 \\ -1.86 & -7.07 & 11.56 & 0.03 \\ 0.07 & -0.71 & 0.03 & 0.01 \end{bmatrix},$$

generated by nag_rand_copula_normal (g05rd). All ten observations are generated by a single call to nag_rand_copula_normal (g05rd) with **mode** $= 2$. The random number generator is initialized by nag_rand_init_repeat (g05kf).

9.1 Program Text

```
function g05rd_example

fprintf('g05rd example results\n\n');

% Initialize the base generator to a repeatable sequence
seed  = [nag_int(1762543)];
genid = nag_int(1);
subid = nag_int(1);
[state, ifail] = g05kf( ...
                        genid, subid, seed);

% Sample size
n  = nag_int(10);

% Upper triangular part of covariance matrix
c = [ 1.69,  0.39, -1.86,  0.07;
      0,    98.01, -7.07, -0.71;
      0,     0,    11.56,  0.03;
      0,     0,     0,     0.01];
m = size(c,1);

% Setup and generate in one go
mode = nag_int(2);

% Generate variates from Gaussian copula
lr = m*(m+1) + 2;
r = zeros(lr, 1);
[r, state, x, ifail] = g05rd( ...
                              mode, n, c, r, state);

disp('Variates');
disp(x);
```

9.2 Program Results

```
g05rd example results

Variates
    0.6364    0.0517    0.4137    0.8817
    0.1065    0.2461    0.7993    0.3806
    0.7460    0.6313    0.2708    0.5421
    0.7983    0.0564    0.6868    0.9234
    0.1046    0.5790    0.8533    0.2208
    0.4925    0.2784    0.3513    0.5158
    0.3843    0.2349    0.9472    0.7801
    0.7871    0.9941    0.9403    0.2044
    0.4982    0.9015    0.7176    0.2914
    0.6717    0.5359    0.5961    0.4487
```