NAG Toolbox

nag_correg_mixeff_ml (g02jb)

1 Purpose

nag_correg_mixeff_ml (g02jb) fits a linear mixed effects regression model using maximum likelihood (ML).

2 Syntax

```
[nff, nrf, df, ml, b, se, gamma, warn, ifail] = nag_correg_mixeff_ml(nvpr,
levels, yvid, fvid, rvid, svid, cwid, vpr, dat, fint, rint, lb, gamma, 'n', n,
'ncol', ncol, 'nfv', nfv, 'nrv', nrv, 'maxit', maxit, 'tol', tol)
```

```
[nff, nrf, df, ml, b, se, gamma, warn, ifail] = g02jb(nvpr, levels, yvid, fvid,
rvid, svid, cwid, vpr, dat, fint, rint, lb, gamma, 'n', n, 'ncol', ncol, 'nfv',
nfv, 'nrv', nrv, 'maxit', maxit, 'tol', tol)
```

Note: the interface to this routine has changed since earlier releases of the toolbox:

At Mark 23: **maxit** and **tol** were made optional

At Mark 22: **n** was made optional.

3 Description

nag_correg_mixeff_ml (g02jb) fits a model of the form:

$$y = X\beta + Z\nu + \epsilon$$

where

y is a vector of n observations on the dependent variable,

X is a known n by p design matrix for the fixed independent variables,

β is a vector of length p of unknown *fixed effects*,

Z is a known n by q design matrix for the random independent variables,

ν is a vector of length q of unknown *random effects*;

and

ϵ is a vector of length n of unknown random errors.

Both ν and ϵ are assumed to have a Gaussian distribution with expectation zero and

$$\mathrm{Var}\begin{bmatrix}\nu\\ \epsilon\end{bmatrix} = \begin{bmatrix}G & 0\\ 0 & R\end{bmatrix}$$

where $R = \sigma_R^2 I$, I is the $n \times n$ identity matrix and G is a diagonal matrix. It is assumed that the random variables, Z, can be subdivided into $g \le q$ groups with each group being identically distributed with expectations zero and variance σ_i^2. The diagonal elements of matrix G therefore take one of the values $\left\{\sigma_i^2 : i = 1, 2, \ldots, g\right\}$, depending on which group the associated random variable belongs to.

The model therefore contains three sets of unknowns, the fixed effects, β, the random effects ν and a vector of $g+1$ variance components, γ, where $\gamma = \left\{\sigma_1^2, \sigma_2^2, \ldots, \sigma_{g-1}^2, \sigma_g^2, \sigma_R^2\right\}$. Rather than working directly with γ, nag_correg_mixeff_ml (g02jb) uses an iterative process to estimate $\gamma^* = \left\{\sigma_1^2/\sigma_R^2, \sigma_2^2/\sigma_R^2, \ldots, \sigma_{g-1}^2/\sigma_R^2, \sigma_g^2/\sigma_R^2, 1\right\}$. Due to the iterative nature of the estimation a set of initial values, γ_0, for γ^* is required. nag_correg_mixeff_ml (g02jb) allows these initial values either to be supplied by you or calculated from the data using the minimum variance quadratic unbiased estimators (MIVQUE0) suggested by Rao (1972).

nag_correg_mixeff_ml (g02jb) fits the model using a quasi-Newton algorithm to maximize the log-likelihood function:

$$-2l_R = \log(|V|) + (n)\log\left(r'V^{-1}r\right) + \log(2\pi/n)$$

where

$$V = ZGZ' + R, \quad r = y - Xb \quad \text{and} \quad b = \left(X'V^{-1}X\right)^{-1}X'V^{-1}y.$$

Once the final estimates for γ^* have been obtained, the value of σ_R^2 is given by:

$$\sigma_R^2 = \left(r'V^{-1}r\right)/(n-p).$$

Case weights, W_c, can be incorporated into the model by replacing $X'X$ and $Z'Z$ with $X'W_cX$ and $Z'W_cZ$ respectively, for a diagonal weight matrix W_c.

The log-likelihood, l_R, is calculated using the sweep algorithm detailed in Wolfinger *et al.* (1994).

4 References

Goodnight J H (1979) A tutorial on the SWEEP operator *The American Statistician* **33(3)** 149–158

Harville D A (1977) Maximum likelihood approaches to variance component estimation and to related problems *JASA* **72** 320–340

Rao C R (1972) Estimation of variance and covariance components in a linear model *J. Am. Stat. Assoc.* **67** 112–115

Stroup W W (1989) Predictable functions and prediction space in the mixed model procedure *Applications of Mixed Models in Agriculture and Related Disciplines* **Southern Cooperative Series Bulletin No. 343** 39–48

Wolfinger R, Tobias R and Sall J (1994) Computing Gaussian likelihoods and their derivatives for general linear mixed models *SIAM Sci. Statist. Comput.* **15** 1294–1310

5 Parameters

5.1 Compulsory Input Parameters

1: **nvpr** – INTEGER

If **rint** $= 1$ and **svid** $\neq 0$, **nvpr** is the number of variance components being estimated – 2, $(g-1)$, else **nvpr** $= g$.

If **nrv** $= 0$, **nvpr** is not referenced.

Constraint: if **nrv** $\neq 0$, $1 \leq$ **nvpr** $\leq$ **nrv**.

2: **levels**(**ncol**) – INTEGER array

levels(i) contains the number of levels associated with the ith variable of the data matrix **dat**. If this variable is continuous or binary (i.e., only takes the values zero or one) then **levels**(i) should be 1; if the variable is discrete then **levels**(i) is the number of levels associated with it and **dat**(j,i) is assumed to take the values 1 to **levels**(i), for $j = 1,2,\ldots,$ **n**.

Constraint: **levels**$(i) \geq 1$, for $i = 1,2,\ldots,$ **ncol**.

3: **yvid** – INTEGER

The column of **dat** holding the dependent, y, variable.

Constraint: $1 \leq$ **yvid** $\leq$ **ncol**.

4: **fvid**(**nfv**) – INTEGER array

The columns of the data matrix **dat** holding the fixed independent variables with **fvid**(i) holding the column number corresponding to the ith fixed variable.

Constraint: $1 \leq$ **fvid**(i) $\leq$ **ncol**, for $i = 1, 2, \ldots,$ **nfv**.

5: **rvid**(**nrv**) – INTEGER array

The columns of the data matrix **dat** holding the random independent variables with **rvid**(i) holding the column number corresponding to the ith random variable.

Constraint: $1 \leq$ **rvid**(i) $\leq$ **ncol**, for $i = 1, 2, \ldots,$ **nrv**.

6: **svid** – INTEGER

The column of **dat** holding the subject variable.

If **svid** $= 0$, no subject variable is used.

Specifying a subject variable is equivalent to specifying the interaction between that variable and all of the random-effects. Letting the notation $Z_1 \times Z_S$ denote the interaction between variables Z_1 and Z_S, fitting a model with **rint** $= 0$, random-effects $Z_1 + Z_2$ and subject variable Z_S is equivalent to fitting a model with random-effects $Z_1 \times Z_S + Z_2 \times Z_S$ and no subject variable. If **rint** $= 1$ the model is equivalent to fitting $Z_S + Z_1 \times Z_S + Z_2 \times Z_S$ and no subject variable.

Constraint: $0 \leq$ **svid** $\leq$ **ncol**.

7: **cwid** – INTEGER

The column of **dat** holding the case weights.

If **cwid** $= 0$, no weights are used.

Constraint: $0 \leq$ **cwid** $\leq$ **ncol**.

8: **vpr**(**nrv**) – INTEGER array

vpr(i) holds a flag indicating the variance of the ith random variable. The variance of the ith random variable is σ_j^2, where $j =$ **vpr**(i) $+ 1$ if **rint** $= 1$ and **svid** $\neq 0$ and $j =$ **vpr**(i) otherwise. Random variables with the same value of j are assumed to be taken from the same distribution.

Constraint: $1 \leq$ **vpr**(i) $\leq$ **nvpr**, for $i = 1, 2, \ldots,$ **nrv**.

9: **dat**($lddat$, **ncol**) – REAL (KIND=nag_wp) array

$lddat$, the first dimension of the array, must satisfy the constraint $lddat \geq$ **n**.

Array containing all of the data. For the ith observation:

> **dat**(i, **yvid**) holds the dependent variable, y;
>
> if **cwid** $\neq 0$, **dat**(i, **cwid**) holds the case weights;
>
> if **svid** $\neq 0$, **dat**(i, **svid**) holds the subject variable.

The remaining columns hold the values of the independent variables.

Constraints:

> if **cwid** $\neq 0$, **dat**(i, **cwid**) ≥ 0.0;
> if **levels**(j) $\neq 1$, $1 \leq$ **dat**(i, j) $\leq$ **levels**(j).

10: **fint** – INTEGER

Flag indicating whether a fixed intercept is included (**fint** $= 1$).

Constraint: **fint** $= 0$ or 1.

11: **rint** – INTEGER

Flag indicating whether a random intercept is included (**rint** $= 1$).

If **svid** $= 0$, **rint** is not referenced.

Constraint: **rint** $= 0$ or 1.

12: **lb** – INTEGER

The size of the array **b**.

Constraint: $\mathbf{lb} \geq \mathbf{fint} + \sum_{i=1}^{\mathbf{nfv}} \max(\mathbf{levels}(\mathbf{fvid}(i)) - 1, 1) + L_S \times \left(\mathbf{rint} + \sum_{i=1}^{\mathbf{nrv}} \mathbf{levels}(\mathbf{rvid}(i))\right)$ where $L_S = \mathbf{levels}(\mathbf{svid})$ if $\mathbf{svid} \neq 0$ and 1 otherwise.

13: **gamma**($\mathbf{nvpr} + \mathbf{2}$) – REAL (KIND=nag_wp) array

Holds the initial values of the variance components, γ_0, with **gamma**(i) the initial value for σ_i^2/σ_R^2, for $i = 1, 2, \ldots, g$. If **rint** $= 1$ and **svid** $\neq 0$, $g =$ **nvpr** $+ 1$, else $g =$ **nvpr**.

If **gamma**$(1) = -1.0$, the remaining elements of **gamma** are ignored and the initial values for the variance components are estimated from the data using MIVQUE0.

Constraint: **gamma**$(1) = -1.0$ or **gamma**$(i) \geq 0.0$, for $i = 1, 2, \ldots, g$.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the first dimension of the array **dat**.

n, the number of observations.

Constraint: **n** ≥ 1.

2: **ncol** – INTEGER

Default: the dimension of the array **levels** and the second dimension of the array **dat**. (An error is raised if these dimensions are not equal.)

The number of columns in the data matrix, **dat**.

Constraint: **ncol** ≥ 1.

3: **nfv** – INTEGER

Default: the dimension of the array **fvid**.

The number of independent variables in the model which are to be treated as being fixed.

Constraint: $0 \leq$ **nfv** $<$ **ncol**.

4: **nrv** – INTEGER

Default: the dimension of the arrays **rvid**, **vpr**. (An error is raised if these dimensions are not equal.)

The number of independent variables in the model which are to be treated as being random.

Constraints:

$0 \leq$ **nrv** $<$ **ncol**;
nrv $+$ **rint** > 0.

5: **maxit** – INTEGER

Default: -1

The maximum number of iterations.

If **maxit** < 0, the default value of 100 is used.

If **maxit** $= 0$, the parameter estimates (β, ν) and corresponding standard errors are calculated based on the value of γ_0 supplied in **gamma**.

6: **tol** – REAL (KIND=nag_wp)

Default: 0

The tolerance used to assess convergence.

If **tol** ≤ 0.0, the default value of $\epsilon^{0.7}$ is used, where ϵ is the ***machine precision***.

5.3 Output Parameters

1: **nff** – INTEGER

The number of fixed effects estimated (i.e., the number of columns, p, in the design matrix X).

2: **nrf** – INTEGER

The number of random effects estimated (i.e., the number of columns, q, in the design matrix Z).

3: **df** – INTEGER

The degrees of freedom.

4: **ml** – REAL (KIND=nag_wp)

$-2l_R(\hat{\gamma})$ where l_R is the log of the maximum likelihood calculated at $\hat{\gamma}$, the estimated variance components returned in **gamma**.

5: **b**(**lb**) – REAL (KIND=nag_wp) array

The parameter estimates, (β, ν), with the first **nff** elements of **b** containing the fixed effect parameter estimates, β and the next **nrf** elements of **b** containing the random effect parameter estimates, ν.

Fixed effects

If **fint** $= 1$, **b**(1) contains the estimate of the fixed intercept. Let L_i denote the number of levels associated with the ith fixed variable, that is $L_i =$ **levels**(**fvid**(i)). Define

if **fint** $= 1$, $F_1 = 2$ else if **fint** $= 0$, $F_1 = 1$;

$F_{i+1} = F_i + \max(L_i - 1, 1)$, $i \geq 1$.

Then for $i = 1, 2, \ldots,$ **nfv**:

if $L_i > 1$, **b**$(F_i + j - 2)$ contains the parameter estimate for the jth level of the ith fixed variable, for $j = 2, 3, \ldots, L_i$;

if $L_i \leq 1$, **b**(F_i) contains the parameter estimate for the ith fixed variable.

Random effects

Redefining L_i to denote the number of levels associated with the ith random variable, that is $L_i =$ **levels**(**rvid**(i)). Define

if **rint** $= 1$, $R_1 = 2$ else if **rint** $= 0$, $R_1 = 1$;
$R_{i+1} = R_i + L_i$, $i \geq 1$.

Then for $i = 1, 2, \ldots,$ **nrv**:

if **svid** $= 0$,

if $L_i > 1$, $\mathbf{b}(\mathbf{nff} + R_i + j - 1)$ contains the parameter estimate for the jth level of the ith random variable, for $j = 1, 2, \ldots, L_i$;

if $L_i \leq 1$, $\mathbf{b}(\mathbf{nff} + R_i)$ contains the parameter estimate for the ith random variable;

if **svid** $\neq 0$,

let L_S denote the number of levels associated with the subject variable, that is $L_S = \mathbf{levels}(\mathbf{svid})$;

if $L_i > 1$, $\mathbf{b}(\mathbf{nff} + (s - 1)L_S + R_i + j - 1)$ contains the parameter estimate for the interaction between the sth level of the subject variable and the jth level of the ith random variable, for $s = 1, 2, \ldots, L_S$ and $j = 1, 2, \ldots, L_i$;

if $L_i \leq 1$, $\mathbf{b}(\mathbf{nff} + (s - 1)L_S + R_i)$ contains the parameter estimate for the interaction between the sth level of the subject variable and the ith random variable, for $s = 1, 2, \ldots, L_S$;

if **rint** $= 1$, $\mathbf{b}(\mathbf{nff} + 1)$ contains the estimate of the random intercept.

6: **se**(**lb**) – REAL (KIND=nag_wp) array

The standard errors of the parameter estimates given in **b**.

7: **gamma**(**nvpr** + **2**) – REAL (KIND=nag_wp) array

gamma(i), for $i = 1, 2, \ldots, g$, holds the final estimate of σ_i^2 and **gamma**($g + 1$) holds the final estimate for σ_R^2.

8: **warn** – INTEGER

Is set to 1 if a variance component was estimated to be a negative value during the fitting process. Otherwise **warn** is set to 0.

If **warn** $= 1$, the negative estimate is set to zero and the estimation process allowed to continue.

9: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, **n** < 2,
or **ncol** < 1,
or $lddat <$ **n**,
or **yvid** < 1 or **yvid** $>$ **ncol**,
or **cwid** < 0 or **cwid** $>$ **ncol**,
or **nfv** < 0 or **nfv** $\geq$ **ncol**,
or **fint** $\neq 0$ and **fint** $\neq 1$,
or **nrv** < 0 or **nrv** $>$ **ncol** or **nrv** + **rint** < 1,
or **nvpr** < 0 or **nvpr** $>$ **nrv**,
or **rint** $\neq 0$ and **rint** $\neq 1$,
or **svid** < 0 or **svid** $>$ **ncol**,
or **lb** is too small.

ifail $= 2$

On entry, **levels**(i) < 1, for at least one i,
or **fvid**(i) < 1, or **fvid**(i) $>$ **ncol**, for at least one i,
or **rvid**(i) < 1, or **rvid**(i) $>$ **ncol**, for at least one i,

or **vpr**$(i) < 1$ or **vpr**$(i) >$ **nvpr**, for at least one i,
or at least one discrete variable in array **dat** has a value greater than that specified in **levels**,
or **gamma**$(i) < 0$, for at least one i, and **gamma**$(1) \neq -1$.

ifail $= 3$

Degrees of freedom < 1. The number of arguments exceed the effective number of observations.

ifail $= 4$

The function failed to converge to the specified tolerance in **maxit** iterations. See Section 9 for advice.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

The accuracy of the results can be adjusted through the use of the **tol** argument.

8 Further Comments

Wherever possible any block structure present in the design matrix Z should be modelled through a subject variable, specified via **svid**, rather than being explicitly entered into **dat**.

nag_correg_mixeff_ml (g02jb) uses an iterative process to fit the specified model and for some problems this process may fail to converge (see **ifail** $= 4$). If the function fails to converge then the maximum number of iterations (see **maxit**) or tolerance (see **tol**) may require increasing; try a different starting estimate in **gamma**. Alternatively, the model can be fit using restricted maximum likelihood (see nag_correg_mixeff_reml (g02ja)) or using the noniterative MIVQUE0.

To fit the model just using MIVQUE0, the first element of **gamma** should be set to -1 and **maxit** should be set to zero.

Although the quasi-Newton algorithm used in nag_correg_mixeff_ml (g02jb) tends to require more iterations before converging compared to the Newton–Raphson algorithm recommended by Wolfinger *et al.* (1994), it does not require the second derivatives of the likelihood function to be calculated and consequentially takes significantly less time per iteration.

9 Example

The following dataset is taken from Stroup (1989) and arises from a balanced split-plot design with the whole plots arranged in a randomized complete block-design.

In this example the full design matrix for the random independent variable, Z, is given by:

$$Z = \begin{pmatrix}
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1
\end{pmatrix}$$

$$= \begin{pmatrix}
A & 0 & 0 & 0 \\
0 & A & 0 & 0 \\
0 & 0 & A & 0 \\
0 & 0 & 0 & A \\
A & 0 & 0 & 0 \\
0 & A & 0 & 0 \\
0 & 0 & A & 0 \\
0 & 0 & 0 & A
\end{pmatrix}, \tag{1}$$

where

$$A = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}.$$

The block structure evident in (1) is modelled by specifying a four-level subject variable, taking the values $\{1,1,1,2,2,2,3,3,3,4,4,4,1,1,1,2,2,2,3,3,3,4,4,4\}$. The first column of 1s is added to A by setting **rint** $= 1$. The remaining columns of A are specified by a three level factor, taking the values, $\{1,2,3,1,2,3,1,\ldots\}$.

9.1 Program Text

```
     function g02jb_example

fprintf('g02jb example results\n\n');

dat = [56, 1, 1, 1, 1;
       50, 1, 2, 1, 1;
       39, 1, 3, 1, 1;
       30, 2, 1, 1, 1;
       36, 2, 2, 1, 1;
       33, 2, 3, 1, 1;
       32, 3, 1, 1, 1;
       31, 3, 2, 1, 1;
       15, 3, 3, 1, 1;
       30, 4, 1, 1, 1;
       35, 4, 2, 1, 1;
```

```
        17, 4, 3, 1, 1;
        41, 1, 1, 2, 1;
        36, 1, 2, 2, 2;
        35, 1, 3, 2, 3;
        25, 2, 1, 2, 1;
        28, 2, 2, 2, 2;
        30, 2, 3, 2, 3;
        24, 3, 1, 2, 1;
        27, 3, 2, 2, 2;
        19, 3, 3, 2, 3;
        25, 4, 1, 2, 1;
        30, 4, 2, 2, 2;
        18, 4, 3, 2, 3];
[n,ncol] = size(dat);

% Number of levels in each variable
levels = [nag_int(1);4;3;2;3];

% Model information
yvid =  nag_int(1);
fvid = [nag_int(3);  4;  5];
rvid = [nag_int(3)];
svid =  nag_int(2);
cwid =  nag_int(0);
fint =  nag_int(1);
rint =  nag_int(1);

% Calculate lb
lb = (rint + sum(levels(rvid)))*prod(levels(svid)) + ...
     fint + sum(levels(fvid)) - numel(fvid);

% Variance component
vpr = [nag_int(1)];
nvpr = nag_int(numel(vpr));

% Initial gamma
gamma = [1; 1; 0];
% Fit the linear mixed effects regresion model
[nff, nrf, df, ml, b, se, gamma, warn, ifail] = ...
g02jb( ...
       nvpr, levels, yvid, fvid, rvid, svid, cwid, vpr, ...
       dat, fint, rint, lb, gamma);

% Display results
if warn
   fprintf(['Warning: At least one variance component was ', ...
            'estimated to be negative and then reset to zero\n\n']);
end
fprintf('Fixed effects (Estimate and Standard Deviation)\n\n');
k = 1;
if fint==1
  fprintf('Intercept%15s%10.4f%10.4f\n', ' ',b(k), se(k));
  k = k + 1;
end
for i = 1:numel(fvid)
  for j = 1:levels(fvid(i))
    if levels(fvid(i))==1 || j>1
      fprintf('Variable %3d Level %3d: %10.4f%10.4f\n', i, j, b(k), se(k));
      k = k + 1;
    end
  end
end
fprintf('\nRandom Effects (Estimate and Standard Deviation\n\n');
if svid==0
  for i = 1:numel(rvid)
    for j = 1:levels(rvid(i))
      fprintf('Variable %4d Level %4d: %10.4f %10.4f\n', i, j, b(k), se(k));
      k = k + 1;
    end
  end
else
```

```
  for l = 1:levels(svid)
    if (rint==1)
      fprintf('Intercept for Subject Level %4d:%12s%10.4f%10.4f\n', ...
              l, ' ', b(k), se(k));
      k = k + 1;
    end
    for i = 1:numel(rvid)
      for j = 1:levels(rvid(i))
        fprintf('Subject Level %4d Variable %4d Level %4d: %10.4f%10.4f\n', ...
                l, i, j, b(k), se(k));
        k = k + 1;
      end
    end
  end
end
fprintf('\n Variance Components\n');
for i = 1:nvpr+rint
  fprintf('%4d%10.4f\n',i,gamma(i));
end
fprintf('\nsigma^2           = %10.4f\n', gamma(nvpr+rint+1));
fprintf('-2log likelihood  = %10.4f\n', ml);
fprintf('DF                = %16d\n', df);
```

9.2 Program Results

```
     g02jb example results

Fixed effects (Estimate and Standard Deviation)

Intercept                  37.0000    4.0421
Variable   1 Level   2:     1.0000    3.0461
Variable   1 Level   3:   -11.0000    3.0461
Variable   2 Level   2:    -8.2500    1.8736
Variable   3 Level   2:     0.5000    2.6497
Variable   3 Level   3:     7.7500    2.6497

Random Effects (Estimate and Standard Deviation

Intercept for Subject Level    1:                10.7631    3.8855
Subject Level    1 Variable    1 Level    1:      3.7276    2.6268
Subject Level    1 Variable    1 Level    2:     -1.4476    2.6268
Subject Level    1 Variable    1 Level    3:      0.3733    2.6268
Intercept for Subject Level    2:                -0.5269    3.8855
Subject Level    2 Variable    1 Level    1:     -3.7171    2.6268
Subject Level    2 Variable    1 Level    2:     -1.2253    2.6268
Subject Level    2 Variable    1 Level    3:      4.8125    2.6268
Intercept for Subject Level    3:                -5.6450    3.8855
Subject Level    3 Variable    1 Level    1:      0.5903    2.6268
Subject Level    3 Variable    1 Level    2:      0.3987    2.6268
Subject Level    3 Variable    1 Level    3:     -2.3806    2.6268
Intercept for Subject Level    4:                -4.5912    3.8855
Subject Level    4 Variable    1 Level    1:     -0.6009    2.6268
Subject Level    4 Variable    1 Level    2:      2.2742    2.6268
Subject Level    4 Variable    1 Level    3:     -2.8052    2.6268

 Variance Components
   1   46.7969
   2   11.5365

sigma^2           =     7.0208
-2log likelihood  =   141.6877
DF                =               16
```