NAG Toolbox

nag_correg_corrmat_nearest (g02aa)

1 Purpose

nag_correg_corrmat_nearest (g02aa) computes the nearest correlation matrix, in the Frobenius norm, to a given square, input matrix.

2 Syntax

```
[g, x, iter, feval, nrmgrd, ifail] = nag_correg_corrmat_nearest(g, 'n', n, 'errtol', errtol, 'maxits', maxits, 'maxit', maxit)
[g, x, iter, feval, nrmgrd, ifail] = g02aa(g, 'n', n, 'errtol', errtol, 'maxits', maxits, 'maxit', maxit)
```

Note: the interface to this routine has changed since earlier releases of the toolbox:

At Mark 23: **errtol**, **maxits** and **maxit** were made optional.

3 Description

A correlation matrix may be characterised as a real square matrix that is symmetric, has a unit diagonal and is positive semidefinite.

nag_correg_corrmat_nearest (g02aa) applies an inexact Newton method to a dual formulation of the problem, as described by Qi and Sun (2006). It applies the improvements suggested by Borsdorf and Higham (2010).

4 References

Borsdorf R and Higham N J (2010) A preconditioned (Newton) algorithm for the nearest correlation matrix *IMA Journal of Numerical Analysis* **30(1)** 94–107

Qi H and Sun D (2006) A quadratically convergent Newton method for computing the nearest correlation matrix *SIAM J. Matrix AnalAppl* **29(2)** 360–385

5 Parameters

5.1 Compulsory Input Parameters

1: **g**$(ldg, \mathbf{n})$ – REAL (KIND=nag_wp) array

ldg, the first dimension of the array, must satisfy the constraint $ldg \geq \mathbf{n}$.

G, the initial matrix.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the first dimension of the array **g** and the second dimension of the array **g**. (An error is raised if these dimensions are not equal.)

The size of the matrix *G*.

Constraint: $\mathbf{n} > 0$.

2: **errtol** – REAL (KIND=nag_wp)

Default: 0.0

The termination tolerance for the Newton iteration. If **errtol** ≤ 0.0 then $\mathbf{n} \times \sqrt{\textit{machine precision}}$ is used.

3: **maxits** – INTEGER

Default: 0

maxits specifies the maximum number of iterations used for the iterative scheme used to solve the linear algebraic equations at each Newton step.

If **maxits** ≤ 0, $2 \times \mathbf{n}$ is used.

4: **maxit** – INTEGER

Default: 0

Specifies the maximum number of Newton iterations.

If **maxit** ≤ 0, 200 is used.

5.3 Output Parameters

1: $\mathbf{g}(ldg, \mathbf{n})$ – REAL (KIND=nag_wp) array

A symmetric matrix $\frac{1}{2}(G + G^{\mathrm{T}})$ with the diagonal set to I.

2: $\mathbf{x}(ldx, \mathbf{n})$ – REAL (KIND=nag_wp) array

Contains the nearest correlation matrix.

3: **iter** – INTEGER

The number of Newton steps taken.

4: **feval** – INTEGER

The number of function evaluations of the dual problem.

5: **nrmgrd** – REAL (KIND=nag_wp)

The norm of the gradient of the last Newton step.

6: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

Constraint: $ldg \geq \mathbf{n}$.

Constraint: $ldx \geq \mathbf{n}$.

Constraint: $\mathbf{n} > 0$.

ifail $= 2$

Newton iteration fails to converge in $\langle value \rangle$ iterations.

ifail $= 3$ (*warning*)

Machine precision is limiting convergence.
The array returned in **x** may still be of interest.

ifail $= 4$

An intermediate eigenproblem could not be solved. This should not occur. Please contact NAG with details of your call.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

The returned accuracy is controlled by **errtol** and limited by ***machine precision***.

8 Further Comments

Arrays are internally allocated by nag_correg_corrmat_nearest (g02aa). The total size of these arrays is $11 \times \mathbf{n} + 3 \times \mathbf{n} \times \mathbf{n} + \max(2 \times \mathbf{n} \times \mathbf{n} + 6 \times \mathbf{n} + 1, 120 + 9 \times \mathbf{n})$ real elements and $5 \times \mathbf{n} + 3$ integer elements.

9 Example

This example finds the nearest correlation matrix to:

$$G = \begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix}$$

9.1 Program Text

```
function g02aa_example

fprintf('g02aa example results\n\n');

g = [ 2, -1,  0,  0;
     -1,  2, -1,  0;
      0, -1,  2, -1;
      0,  0, -1,  2];

[g, x, iter, feval, nrmgrd, ifail] = ...
  g02aa(g);

fprintf('\n     Nearest Correlation Matrix\n');
disp(x);
fprintf('\n Number of Newton steps taken:   %d\n', iter);
fprintf(' Number of function evaluations: %d\n', feval);
if (nrmgrd > 4*x02aj)
  fprintf(' Norm of gradient of last Newton step: %6.4f\n', nrmgrd);
end
```

9.2 Program Results

```
     g02aa example results

     Nearest Correlation Matrix
    1.0000   -0.8084    0.1916    0.1068
   -0.8084    1.0000   -0.6562    0.1916
    0.1916   -0.6562    1.0000   -0.8084
    0.1068    0.1916   -0.8084    1.0000

 Number of Newton steps taken:   3
 Number of function evaluations: 4
 Norm of gradient of last Newton step: 0.0000
```