NAG Toolbox

nag_stat_pdf_multi_normal_vector (g01lb)

1 Purpose

nag_stat_pdf_multi_normal_vector (g01lb) returns a number of values of the probability density function (PDF), or its logarithm, for the multivariate Normal (Gaussian) distribution.

2 Syntax

```
[pdf, rank, ifail] = nag_stat_pdf_multi_normal_vector(ilog, k, x, xmu, iuld, sig, 'n', n)
[pdf, rank, ifail] = g01lb(ilog, k, x, xmu, iuld, sig, 'n', n)
```

Note: the interface to this routine has changed since earlier releases of the toolbox:

At Mark 25: **k** was made optional.

3 Description

The probability density function, $f(X : \mu, \Sigma)$ of an n-dimensional multivariate Normal distribution with mean vector μ and n by n variance-covariance matrix Σ, is given by

$$f(X : \mu, \Sigma) = ((2\pi)^n |\Sigma|)^{-1/2} \exp\left(-\tfrac{1}{2}(X-\mu)^{\mathrm{T}} \Sigma^{-1} (X-\mu)\right).$$

If the variance-covariance matrix, Σ, is not of full rank then the probability density function, is calculated as

$$f(X : \mu, \Sigma) = ((2\pi)^r \mathrm{pdet}(\Sigma))^{-1/2} \exp\left(-\tfrac{1}{2}(X-\mu)^{\mathrm{T}} \Sigma^{-} (X-\mu)\right)$$

where $\mathrm{pdet}(\Sigma)$ is the pseudo-determinant, Σ^{-} a generalized inverse of Σ and r its rank.

nag_stat_pdf_multi_normal_vector (g01lb) evaluates the PDF at k points with a single call.

4 References

None.

5 Parameters

5.1 Compulsory Input Parameters

1: **ilog** – INTEGER

The value of **ilog** determines whether the logarithmic value is returned in PDF.

ilog $= 0$
$f(X : \mu, \Sigma)$, the probability density function is returned.

ilog $= 1$
$\log(f(X : \mu, \Sigma))$, the logarithm of the probability density function is returned.

Constraint: **ilog** $= 0$ or 1.

2: **k** – INTEGER

k, the number of points the PDF is to be evaluated at.

Constraint: **k** ≥ 0.

3: **x**$(ldx, :)$ – REAL (KIND=nag_wp) array

The first dimension of the array **x** must be at least **n**.

The second dimension of the array **x** must be at least **k**.

X, the matrix of k points at which to evaluate the probability density function, with the ith dimension for the jth point held in **x**(i, j).

4: **xmu**(**n**) – REAL (KIND=nag_wp) array

μ, the mean vector of the multivariate Normal distribution.

5: **iuld** – INTEGER

Indicates the form of Σ and how it is stored in **sig**.

iuld $= 1$
: **sig** holds the lower triangular portion of Σ.

iuld $= 2$
: **sig** holds the upper triangular portion of Σ.

iuld $= 3$
: Σ is a diagonal matrix and **sig** only holds the diagonal elements.

iuld $= 4$
: **sig** holds the lower Cholesky decomposition, L such that $LL^{\mathrm{T}} = \Sigma$.

iuld $= 5$
: **sig** holds the upper Cholesky decomposition, U such that $U^{\mathrm{T}}U = \Sigma$.

Constraint: **iuld** $= 1$, 2, 3, 4 or 5.

6: **sig**$(ldsig, :)$ – REAL (KIND=nag_wp) array

The first dimension, $ldsig$, of the array **sig** must satisfy

if **iuld** $= 3$, $ldsig \geq 1$;
otherwise $ldsig \geq$ **n**.

The second dimension of the array **sig** must be at least **n**.

Information defining the variance-covariance matrix, Σ.

iuld $= 1$ or 2
: **sig** must hold the lower or upper portion of Σ, with Σ_{ij} held in **sig**(i, j). The supplied variance-covariance matrix must be positive semidefinite.

iuld $= 3$
: Σ is a diagonal matrix and the ith diagonal element, Σ_{ii}, must be held in **sig**$(1, i)$

iuld $= 4$ or 5
: **sig** must hold L or U, the lower or upper Cholesky decomposition of Σ, with L_{ij} or U_{ij} held in **sig**(i, j), depending on the value of **iuld**. No check is made that LL^{T} or $U^{\mathrm{T}}U$ is a valid variance-covariance matrix. The diagonal elements of the supplied L or U must be greater than zero

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the first dimension of the array **x** and the dimension of the array **xmu** and the first dimension of the array **sig** and the second dimension of the array **sig**. (An error is raised if these dimensions are not equal.)

n, the number of dimensions.

Constraint: $\mathbf{n} \geq 2$.

5.3 Output Parameters

1: **pdf**(**k**) – REAL (KIND=nag_wp) array

$f(X : \mu, \Sigma)$ or $\log(f(X : \mu, \Sigma))$ depending on the value of **ilog**.

2: **rank** – INTEGER

r, rank of Σ.

3: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 11$

Constraint: **ilog** $= 0$ or 1.

ifail $= 21$

Constraint: $\mathbf{k} \geq 0$.

ifail $= 31$

Constraint: $\mathbf{n} \geq 2$.

ifail $= 51$

Constraint: $ldx \geq \mathbf{n}$.

ifail $= 71$

Constraint: **iuld** $= 1, 2, 3, 4$ or 5.

ifail $= 81$

On entry, Σ is not positive semidefinite.

ifail $= 82$

On entry, at least one diagonal element of Σ is less than or equal to 0.

ifail $= 83$

On entry, Σ is not positive definite and eigenvalue decomposition failed.

ifail $= 91$

Constraint: if **iuld** $= 3$, $ldsig \geq 1$.

ifail $= 92$

Constraint: if **iuld** $\neq 3$, $ldsig \geq$ **n**.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

Not applicable.

8 Further Comments

None.

9 Example

This example prints the value of the multivariate Normal PDF at a number of different points.

9.1 Program Text

```
function g01lb_example

fprintf('g01lb example results\n\n');

ilog = nag_int(0);
k    = nag_int(2);
iuld = nag_int(1);
xmu  = [0.1; 0.2; 0.3; 0.4];
sig  = [4.16,  0,     0,     0;
       -3.12,  5.03,  0,     0;
        0.56, -0.83,  0.76,  0;
       -0.10,  1.18,  0.34,  1.18];
x    = [1, 1;
        1, 2;
        1, 3;
        1, 4];

[pdf, rnk, ifail] = g01lb(ilog, x, xmu, iuld, sig);

fprintf('\nRank of the covariance matrix: %d\n', rnk);
if ilog == 1
  fprintf('     log(PDF)                  X\n');
else
  fprintf('       PDF                     X\n');
end
for i=1:k
  fprintf('%13.4e %8.4f %8.4f %8.4f %8.4f\n', pdf(i), x(:, i));
end
```

9.2 Program Results

```
     g01lb example results

Rank of the covariance matrix: 4
       PDF                    X
   3.0307e-03   1.0000   1.0000   1.0000   1.0000
   4.5232e-06   1.0000   2.0000   3.0000   4.0000
```