NAG Toolbox

nag_stat_prob_chisq_noncentral (g01gc)

1 Purpose

nag_stat_prob_chisq_noncentral (g01gc) returns the probability associated with the lower tail of the noncentral χ^2-distribution via the function name.

2 Syntax

```
[result, ifail] = nag_stat_prob_chisq_noncentral(x, df, rlamda, 'tol', tol, 'maxit', maxit)
[result, ifail] = g01gc(x, df, rlamda, 'tol', tol, 'maxit', maxit)
```

Note: the interface to this routine has changed since earlier releases of the toolbox:

At Mark 23: **tol** was made optional (default 0).

3 Description

The lower tail probability of the noncentral χ^2-distribution with ν degrees of freedom and noncentrality parameter λ, $P(X \leq x : \nu;\lambda)$, is defined by

$$P(X \leq x : \nu;\lambda) = \sum_{j=0}^{\infty} e^{-\lambda/2} \frac{(\lambda/2)^j}{j!} P(X \leq x : \nu + 2j;0), \tag{1}$$

where $P(X \leq x : \nu + 2j;0)$ is a central χ^2-distribution with $\nu + 2j$ degrees of freedom.

The value of j at which the Poisson weight, $e^{-\lambda/2}\frac{(\lambda/2)^j}{j!}$, is greatest is determined and the summation (1) is made forward and backward from that value of j.

The recursive relationship:

$$P(X \leq x : a + 2;0) = P(X \leq x : a;0) - \frac{(x^a/2)e^{-x/2}}{\Gamma(a+1)} \tag{2}$$

is used during the summation in (1).

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

5.1 Compulsory Input Parameters

1: **x** – REAL (KIND=nag_wp)

The deviate from the noncentral χ^2-distribution with ν degrees of freedom and noncentrality parameter λ.

Constraint: $\mathbf{x} \geq 0.0$.

2: **df** – REAL (KIND=nag_wp)

ν, the degrees of freedom of the noncentral χ^2-distribution.

Constraint: **df** ≥ 0.0.

3: **rlamda** – REAL (KIND=nag_wp)

λ, the noncentrality parameter of the noncentral χ^2-distribution.

Constraint: **rlamda** ≥ 0.0 if **df** > 0.0 or **rlamda** > 0.0 if **df** $= 0.0$.

5.2 Optional Input Parameters

1: **tol** – REAL (KIND=nag_wp)

Default: 0.0

The required accuracy of the solution. If nag_stat_prob_chisq_noncentral (g01gc) is entered with **tol** greater than or equal to 1.0 or less than $10 \times$ ***machine precision*** (see nag_machine_precision (x02aj)), then the value of $10 \times$ ***machine precision*** is used instead.

2: **maxit** – INTEGER

Default: 100. See Section 9 for further discussion.

The maximum number of iterations to be performed.

Constraint: **maxit** ≥ 1.

5.3 Output Parameters

1: **result**

The result of the function.

2: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Note: nag_stat_prob_chisq_noncentral (g01gc) may return useful information for one or more of the following detected errors or warnings.

Errors or warnings detected by the function:

If on exit **ifail** $= 1$, 2, 4 or 5, then nag_stat_prob_chisq_noncentral (g01gc) returns 0.0.

ifail $= 1$

On entry, **df** < 0.0,
or **rlamda** < 0.0,
or **df** $= 0.0$ and **rlamda** $= 0.0$,
or **x** < 0.0,
or **maxit** < 1.

ifail $= 2$

The initial value of the Poisson weight used in the summation (1) was too small to be calculated. The value of $P(\mathbf{x} \leq x : \nu;\lambda)$ is likely to be zero.

ifail $= 3$

The solution has failed to converge in **maxit** iterations.

ifail $= 4$

The value of a term required in (2) is too large to be evaluated accurately. The most likely cause of this error is both **x** and **rlamda** being very large.

ifail $= 5$

The calculations for the central χ^2 probability has failed to converge. This is an unlikely error exit. A larger value of **tol** should be used.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

The summations described in Section 3 are made until an upper bound on the truncation error relative to the current summation value is less than **tol**.

8 Further Comments

The number of terms in (1) required for a given accuracy will depend on the following factors:

(i) The rate at which the Poisson weights tend to zero. This will be slower for larger values of λ.

(ii) The rate at which the central χ^2 probabilities tend to zero. This will be slower for larger values of ν and x.

9 Example

This example reads values from various noncentral χ^2-distributions, calculates the lower tail probabilities and prints all these values until the end of data is reached.

9.1 Program Text

```
function g01gc_example

fprintf('g01gc example results\n\n');

x      = [  8.26   6.2   55.76];
df     = [ 20      7.5   45   ];
rlamda = [  3.5    2      1   ];
p      = x;

fprintf('     x       df   rlamda     p\n');
for j = 1:numel(x)
   [p(j), ifail] = g01gc( ...
   x(j), df(j), rlamda(j));
end

fprintf('%8.3f%8.3f%8.3f%8.4f\n', [x; df; rlamda; p]);
```

9.2 Program Results

```
     g01gc example results

     x       df   rlamda     p
   8.260  20.000   3.500  0.0032
   6.200   7.500   2.000  0.2699
  55.760  45.000   1.000  0.8443
```