NAG Toolbox

nag_stat_prob_students_t_noncentral (g01gb)

1 Purpose

nag_stat_prob_students_t_noncentral (g01gb) returns the lower tail probability for the noncentral Student's t-distribution.

2 Syntax

```
[result, ifail] = nag_stat_prob_students_t_noncentral(t, df, delta, 'tol', tol, 'maxit', maxit)
[result, ifail] = g01gb(t, df, delta, 'tol', tol, 'maxit', maxit)
```

Note: the interface to this routine has changed since earlier releases of the toolbox:

At Mark 23: **tol** was made optional (default 0).

3 Description

The lower tail probability of the noncentral Student's t-distribution with ν degrees of freedom and noncentrality parameter δ, $P(T \leq t : \nu;\delta)$, is defined by

$$P(T \leq t : \nu;\delta) = C_\nu \int_0^\infty \left(\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\alpha u - \delta} e^{-x^2/2}\, dx \right) u^{\nu-1} e^{-u^2/2} du, \quad \nu > 0.0$$

with

$$C_\nu = \frac{1}{\Gamma\left(\frac{1}{2}\nu\right) 2^{(\nu-2)/2}}, \quad \alpha = \frac{t}{\sqrt{\nu}}.$$

The probability is computed in one of two ways.

(i) When $t = 0.0$, the relationship to the normal is used:

$$P(T \leq t : \nu;\delta) = \frac{1}{\sqrt{2\pi}} \int_\delta^\infty e^{-u^2/2}\, du.$$

(ii) Otherwise the series expansion described in Equation 9 of Amos (1964) is used. This involves the sums of confluent hypergeometric functions, the terms of which are computed using recurrence relationships.

4 References

Amos D E (1964) Representations of the central and non-central t-distributions *Biometrika* **51** 451–458

5 Parameters

5.1 Compulsory Input Parameters

1: **t** – REAL (KIND=nag_wp)

t, the deviate from the Student's t-distribution with ν degrees of freedom.

2: **df** – REAL (KIND=nag_wp)

ν, the degrees of freedom of the Student's t-distribution.

Constraint: **df** ≥ 1.0.

3: **delta** – REAL (KIND=nag_wp)

δ, the noncentrality argument of the Students t-distribution.

5.2 Optional Input Parameters

1: **tol** – REAL (KIND=nag_wp)

Default: 0.0

The absolute accuracy required by you in the results. If nag_stat_prob_students_t_noncentral (g01gb) is entered with **tol** greater than or equal to 1.0 or less than $10 \times$ ***machine precision*** (see nag_machine_precision (x02aj)), then the value of $10 \times$ ***machine precision*** is used instead.

2: **maxit** – INTEGER

Default: 100. See Section 9 for further comments.

The maximum number of terms that are used in each of the summations.

Constraint: **maxit** ≥ 1.

5.3 Output Parameters

1: **result**

The result of the function.

2: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

If on exit **ifail** $\neq 0$, then nag_stat_prob_students_t_noncentral (g01gb) returns 0.0.

ifail $= 1$

On entry, **df** < 1.0.

ifail $= 2$

On entry, **maxit** < 1.

ifail $= 3$

One of the series has failed to converge. Reconsider the requested tolerance and/or maximum number of iterations.

ifail $= 4$

The probability is too small to calculate accurately.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

The series described in Amos (1964) are summed until an estimated upper bound on the contribution of future terms to the probability is less than **tol**. There may also be some loss of accuracy due to calculation of gamma functions.

8 Further Comments

The rate of convergence of the series depends, in part, on the quantity $t^2/\left(t^2+\nu\right)$. The smaller this quantity the faster the convergence. Thus for large t and small ν the convergence may be slow. If ν is an integer then one of the series to be summed is of finite length.

If two tail probabilities are required then the relationship of the t-distribution to the F-distribution can be used:

$$F = T^2, \lambda = \delta^2, \nu_1 = 1 \quad \text{and} \quad \nu_2 = \nu,$$

and a call made to nag_stat_prob_f_noncentral (g01gd).

Note that nag_stat_prob_students_t_noncentral (g01gb) only allows degrees of freedom greater than or equal to 1 although values between 0 and 1 are theoretically possible.

9 Example

This example reads values from, and degrees of freedom for, and noncentrality arguments of the noncentral Student's t-distributions, calculates the lower tail probabilities and prints all these values until the end of data is reached.

9.1 Program Text

```
     function g01gb_example

fprintf('g01gb example results\n\n');

t     = [ -1.528   -0.188   1.138];
df    = [ 20        7.5    45    ];
delta = [  2        1       0    ];
p     = t;

fprintf('     t       df    delta     p\n');
for j = 1:numel(t)
   [p(j), ifail] = g01gb( ...
   t(j), df(j), delta(j));
end

fprintf('%8.3f%8.3f%8.3f%8.4f\n', [t; df; delta; p]);
```

9.2 Program Results

```
     g01gb example results

     t       df    delta     p
  -1.528  20.000   2.000  0.0003
  -0.188   7.500   1.000  0.1189
   1.138  45.000   0.000  0.8694
```