NAG Toolbox

nag_stat_inv_cdf_studentized_range (g01fm)

1 Purpose

nag_stat_inv_cdf_studentized_range (g01fm) returns the deviate associated with the lower tail probability of the distribution of the Studentized range statistic.

2 Syntax

```
[result, ifail] = nag_stat_inv_cdf_studentized_range(p, v, ir)
[result, ifail] = g01fm(p, v, ir)
```

3 Description

The externally Studentized range, q, for a sample, $x_1, x_2, \ldots, x_r$, is defined as

$$q = \frac{\max(x_i) - \min(x_i)}{\hat{\sigma}_e},$$

where $\hat{\sigma}_e$ is an independent estimate of the standard error of the x_i. The most common use of this statistic is in the testing of means from a balanced design. In this case for a set of group means, $\bar{T}_1, \bar{T}_2, \ldots, \bar{T}_r$, the Studentized range statistic is defined to be the difference between the largest and smallest means, $\bar{T}_{\text{largest}}$ and $\bar{T}_{\text{smallest}}$, divided by the square root of the mean-square experimental error, MS_{error}, over the number of observations in each group, n, i.e.,

$$q = \frac{\bar{T}_{\text{largest}} - \bar{T}_{\text{smallest}}}{\sqrt{MS_{\text{error}}/n}}.$$

The Studentized range statistic can be used as part of a multiple comparisons procedure such as the Newman–Keuls procedure or Duncan's multiple range test (see Montgomery (1984) and Winer (1970)).

For a Studentized range statistic the probability integral, $P(q; v, r)$, for v degrees of freedom and r groups, can be written as:

$$P(q; v, r) = C \int_0^\infty x^{v-1} e^{-vx^2/2} \left(r \int_{-\infty}^\infty \phi(y) (\Phi(y) - \Phi(y - qx))^{r-1}\, dy \right) dx,$$

where

$$C = \frac{v^{v/2}}{\Gamma(v/2) 2^{v/2-1}}, \quad \phi(y) = \frac{1}{\sqrt{2\pi}} e^{-y^2/2} \quad \text{and} \quad \Phi(y) = \int_{-\infty}^y \phi(t)\, dt.$$

For a given probability p_0, the deviate q_0 is found as the solution to the equation

$$P(q_0; v, r) = p_0, \tag{1}$$

using nag_roots_contfn_brent_rcomm (c05az) . Initial estimates are found using the approximation given in Lund and Lund (1983) and a simple search procedure.

4 References

Lund R E and Lund J R (1983) Algorithm AS 190: probabilities and upper quartiles for the studentized range *Appl. Statist.* **32(2)** 204–210

Montgomery D C (1984) *Design and Analysis of Experiments* Wiley

Winer B J (1970) *Statistical Principles in Experimental Design* McGraw–Hill

5 Parameters

5.1 Compulsory Input Parameters

1: **p** – REAL (KIND=nag_wp)

The lower tail probability for the Studentized range statistic, p_0.

Constraint: $0.0 < \mathbf{p} < 1.0$.

2: **v** – REAL (KIND=nag_wp)

v, the number of degrees of freedom.

Constraint: $\mathbf{v} \geq 1.0$.

3: **ir** – INTEGER

r, the number of groups.

Constraint: $\mathbf{ir} \geq 2$.

5.2 Optional Input Parameters

None.

5.3 Output Parameters

1: **result**

The result of the function.

2: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Note: nag_stat_inv_cdf_studentized_range (g01fm) may return useful information for one or more of the following detected errors or warnings.

Errors or warnings detected by the function:

If on exit **ifail** $= 1$, then nag_stat_inv_cdf_studentized_range (g01fm) returns 0.0.

ifail $= 1$

On entry, $\mathbf{p} \leq 0.0$,
or $\mathbf{p} \geq 1.0$,
or $\mathbf{v} < 1.0$,
or $\mathbf{ir} < 2$.

ifail $= 2$

The function was unable to find an upper bound for the value of q_0. This will be caused by p_0 being too close to 1.0.

ifail $= 3$ (*warning*)

There is some doubt as to whether full accuracy has been achieved. The returned value should be a reasonable estimate of the true value.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

The returned solution, q_*, to equation (1) is determined so that at least one of the following criteria apply.

(a) $|P(q_*;v,r) - p_0| \leq 0.000005$

(b) $|q_0 - q_*| \leq 0.000005 \times \max(1.0, |q_*|)$.

8 Further Comments

To obtain the factors for Duncan's multiple-range test, equation (1) has to be solved for p_1, where $p_1 = p_0^{r-1}$, so on input **p** should be set to p_0^{r-1}.

9 Example

Three values of p, ν and r are read in and the Studentized range deviates or quantiles are computed and printed.

9.1 Program Text

```
function g01fm_example

fprintf('g01fm example results\n\n');

p  = [     0.95      0.30    0.9];
v  = [   10        60        5  ];
ir = [nag_int(5)  12        4  ];
quantile  = p;

fprintf('     p        v     ir   quantile\n');
for j = 1:numel(p)
   [quantile(j), ifail] = g01fm( ...
  p(j), v(j), ir(j));
end

fprintf('%8.3f%8.3f%4d %8.3f\n', [p; v; double(ir); quantile]);
```

9.2 Program Results

```
g01fm example results

    p        v     ir   quantile
  0.950  10.000   5     4.654
  0.300  60.000  12     2.810
  0.900   5.000   4     4.264
```