NAG Toolbox

nag_stat_inv_cdf_gamma (g01ff)

1 Purpose

nag_stat_inv_cdf_gamma (g01ff) returns the deviate associated with the given lower tail probability of the gamma distribution.

2 Syntax

```
[result, ifail] = nag_stat_inv_cdf_gamma(p, a, b, 'tol', tol)
[result, ifail] = g01ff(p, a, b, 'tol', tol)
```

Note: the interface to this routine has changed since earlier releases of the toolbox:

At Mark 23: **tol** was made optional (default 0).

3 Description

The deviate, g_p, associated with the lower tail probability, p, of the gamma distribution with shape parameter α and scale parameter β, is defined as the solution to

$$P\big(G \leq g_p : \alpha, \beta\big) = p = \frac{1}{\beta^{\alpha}\Gamma(\alpha)}\int_0^{g_p} e^{-G/\beta}G^{\alpha-1}\,dG, \quad 0 \leq g_p < \infty; \alpha, \beta > 0.$$

The method used is described by Best and Roberts (1975) making use of the relationship between the gamma distribution and the χ^2-distribution.

Let $y = 2\frac{g_p}{\beta}$. The required y is found from the Taylor series expansion

$$y = y_0 + \sum_r \frac{C_r(y_0)}{r!}\left(\frac{E}{\phi(y_0)}\right)^r,$$

where y_0 is a starting approximation

$$C_1(u) = 1,$$

$$C_{r+1}(u) = \left(r\Psi + \frac{d}{du}\right)C_r(u),$$

$$\Psi = \tfrac{1}{2} - \frac{\alpha - 1}{u},$$

$$E = p - \int_0^{y_0} \phi(u)\,du,$$

$$\phi(u) = \frac{1}{2^{\alpha}\Gamma(\alpha)}e^{-u/2}u^{\alpha-1}.$$

For most values of p and α the starting value

$$y_{01} = 2\alpha\left(z\sqrt{\frac{1}{9\alpha}} + 1 - \frac{1}{9\alpha}\right)^3$$

is used, where z is the deviate associated with a lower tail probability of p for the standard Normal distribution.

For p close to zero,

$$y_{02} = (p\alpha 2^{\alpha}\Gamma(\alpha))^{1/\alpha}$$

is used.

For large p values, when $y_{01} > 4.4\alpha + 6.0$,

$$y_{03} = -2\left[\ln(1-p) - (\alpha-1)\ln\left(\tfrac{1}{2}y_{01}\right) + \ln(\Gamma(\alpha))\right]$$

is found to be a better starting value than y_{01}.

For small α ($\alpha \leq 0.16$), p is expressed in terms of an approximation to the exponential integral and y_{04} is found by Newton–Raphson iterations.

Seven terms of the Taylor series are used to refine the starting approximation, repeating the process if necessary until the required accuracy is obtained.

4 References

Best D J and Roberts D E (1975) Algorithm AS 91. The percentage points of the χ^2 distribution *Appl. Statist.* **24** 385–388

5 Parameters

5.1 Compulsory Input Parameters

1: **p** – REAL (KIND=nag_wp)

p, the lower tail probability from the required gamma distribution.

Constraint: $0.0 \leq \mathbf{p} < 1.0$.

2: **a** – REAL (KIND=nag_wp)

α, the shape parameter of the gamma distribution.

Constraint: $0.0 < \mathbf{a} \leq 10^6$.

3: **b** – REAL (KIND=nag_wp)

β, the scale parameter of the gamma distribution.

Constraint: $\mathbf{b} > 0.0$.

5.2 Optional Input Parameters

1: **tol** – REAL (KIND=nag_wp)

Default: 0.0

The relative accuracy required by you in the results. The smallest recommended value is $50 \times \delta$, where $\delta = \max\left(10^{-18}, \textbf{\textit{machine precision}}\right)$. If nag_stat_inv_cdf_gamma (g01ff) is entered with **tol** less than $50 \times \delta$ or greater or equal to 1.0, then $50 \times \delta$ is used instead.

5.3 Output Parameters

1: **result**

The result of the function.

2: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Note: nag_stat_inv_cdf_gamma (g01ff) may return useful information for one or more of the following detected errors or warnings.

Errors or warnings detected by the function:

If on exit **ifail** $= 1$, 2, 3 or 5, then nag_stat_inv_cdf_gamma (g01ff) returns 0.0.

ifail $= 1$

On entry, **p** < 0.0,
or **p** ≥ 1.0,

ifail $= 2$

On entry, **a** ≤ 0.0,
or **a** $> 10^6$,
or **b** ≤ 0.0

ifail $= 3$

p is too close to 0.0 or 1.0 to enable the result to be calculated.

ifail $= 4$ (*warning*)

The solution has failed to converge in 100 iterations. A larger value of **tol** should be tried. The result may be a reasonable approximation.

ifail $= 5$

The series to calculate the gamma function has failed to converge. This is an unlikely error exit.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

In most cases the relative accuracy of the results should be as specified by **tol**. However, for very small values of α or very small values of p there may be some loss of accuracy.

8 Further Comments

None.

9 Example

This example reads lower tail probabilities for several gamma distributions, and calculates and prints the corresponding deviates until the end of data is reached.

9.1 Program Text

```
function g01ff_example

fprintf('g01ff example results\n\n');

p = [  0.01    0.428   0.869 ];
a = [  1       7.5    45     ];
b = [ 20       0.1    10     ];
x  = p;

fprintf('     p       a       b       x\n');
for j = 1:numel(p)
   [x(j), ifail] = g01ff( ...
     p(j), a(j), b(j));
end

fprintf('%8.3f%8.3f%8.3f%8.3f\n', [p; a; b; x]);
```

9.2 Program Results

```
g01ff example results

     p       a       b       x
   0.010   1.000  20.000   0.201
   0.428   7.500   0.100   0.670
   0.869  45.000  10.000 525.839
```