NAG Toolbox

nag_stat_inv_cdf_students_t (g01fb)

1 Purpose

nag_stat_inv_cdf_students_t (g01fb) returns the deviate associated with the given tail probability of Student's t-distribution with real degrees of freedom.

2 Syntax

```
[result, ifail] = nag_stat_inv_cdf_students_t(p, df, 'tail', tail)
[result, ifail] = g01fb(p, df, 'tail', tail)
```

Note: the interface to this routine has changed since earlier releases of the toolbox:

At Mark 23: **tail** was made optional (default 'L').

3 Description

The deviate, t_p associated with the lower tail probability, p, of the Student's t-distribution with ν degrees of freedom is defined as the solution to

$$P\left(T < t_p : \nu\right) = p = \frac{\Gamma((\nu+1)/2)}{\sqrt{\nu\pi}\Gamma(\nu/2)} \int_{-\infty}^{t_p} \left(1 + \frac{T^2}{\nu}\right)^{-(\nu+1)/2} dT, \quad \nu \geq 1; \; -\infty < t_p < \infty.$$

For $\nu = 1$ or 2 the integral equation is easily solved for t_p.

For other values of $\nu < 3$ a transformation to the beta distribution is used and the result obtained from nag_stat_inv_cdf_beta (g01fe).

For $\nu \geq 3$ an inverse asymptotic expansion of Cornish–Fisher type is used. The algorithm is described by Hill (1970).

4 References

Hastings N A J and Peacock J B (1975) *Statistical Distributions* Butterworth

Hill G W (1970) Student's t-distribution *Comm. ACM* **13(10)** 617–619

5 Parameters

5.1 Compulsory Input Parameters

1: **p** – REAL (KIND=nag_wp)

p, the probability from the required Student's t-distribution as defined by **tail**.

Constraint: $0.0 < \mathbf{p} < 1.0$.

2: **df** – REAL (KIND=nag_wp)

ν, the degrees of freedom of the Student's t-distribution.

Constraint: $\mathbf{df} \geq 1.0$.

5.2 Optional Input Parameters

1: **tail** – CHARACTER(1)

Default: 'L'

Indicates which tail the supplied probability represents.

tail = 'U'
The upper tail probability, i.e., $P\left(T \geq t_p : \nu\right)$.

tail = 'L'
The lower tail probability, i.e., $P\left(T \leq t_p : \nu\right)$.

tail = 'S'
The two tail (significance level) probability, i.e., $P\left(T \geq \left|t_p\right| : \nu\right) + P\left(T \leq -\left|t_p\right| : \nu\right)$.

tail = 'C'
The two tail (confidence interval) probability, i.e., $P\left(T \leq \left|t_p\right| : \nu\right) - P\left(T \leq -\left|t_p\right| : \nu\right)$.

Constraint: **tail** = 'U', 'L', 'S' or 'C'.

5.3 Output Parameters

1: **result**

The result of the function.

2: **ifail** – INTEGER

ifail = 0 unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Note: nag_stat_inv_cdf_students_t (g01fb) may return useful information for one or more of the following detected errors or warnings.

Errors or warnings detected by the function:

If **ifail** = 1, 2 or 3 on exit, then nag_stat_inv_cdf_students_t (g01fb) returns zero.

ifail = 1

On entry, **tail** $\neq$ 'U', 'S', 'C' or 'L'.

ifail = 2

On entry, **p** ≤ 0.0,
or **p** ≥ 1.0.

ifail = 3

On entry, **df** < 1.0.

ifail = 5 (*warning*)

Convergence in the calculation of the inverse beta value was not achieved. However, the result should be a reasonable approximation to the correct value.

ifail = -99

An unexpected error has been triggered by this routine. Please contact NAG.

ifail = -399

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

The results should be accurate to five significant digits, for most argument values. The error behaviour for various argument values is discussed in Hill (1970).

8 Further Comments

The value t_p may be calculated by using the transformation described in Section 3 and using nag_stat_inv_cdf_beta (g01fe). This function allows you to set the required accuracy.

9 Example

This example reads the probability, the tail that probability represents and the degrees of freedom for a number of Student's t-distributions and computes the corresponding deviates.

9.1 Program Text

```
     function g01fb_example

fprintf('g01fb example results\n\n');

p    = [ 0.01;          0.01;        0.99];
df   = [20;             7.5;        45.00];
tail = {'Significance'; 'Lower'; 'Confidence' };

fprintf('      p      df    tail      x\n');
for j = 1:numel(p);

  [x, ifail] = g01fb( ...
                      p(j) , df(j), 'tail', tail{j});

  fprintf('%9.3f%8.3f%4s%12.3f\n', p(j), df(j), tail{j}(1), x);
end
```

9.2 Program Results

```
     g01fb example results

      p      df    tail      x
    0.010  20.000   S       2.845
    0.010   7.500   L      -2.943
    0.990  45.000   C       2.690
```