

NAG Toolbox

nag_stat_prob_studentized_range (g01em)

1 Purpose

nag_stat_prob_studentized_range (g01em) returns the probability associated with the lower tail of the distribution of the Studentized range statistic.

2 Syntax

```
[result, ifail] = nag_stat_prob_studentized_range(q, v, ir)
```

```
[result, ifail] = g01em(q, v, ir)
```

3 Description

The externally Studentized range, q , for a sample, x_1, x_2, \dots, x_r , is defined as:

$$q = \frac{\max x_i - \min x_i}{\hat{\sigma}_e},$$

where $\hat{\sigma}_e$ is an independent estimate of the standard error of the x_i 's. The most common use of this statistic is in the testing of means from a balanced design. In this case for a set of group means, $\bar{T}_1, \bar{T}_2, \dots, \bar{T}_r$, the Studentized range statistic is defined to be the difference between the largest and smallest means, \bar{T}_{largest} and $\bar{T}_{\text{smallest}}$, divided by the square root of the mean-square experimental error, MS_{error} , over the number of observations in each group, n , i.e.,

$$q = \frac{\bar{T}_{\text{largest}} - \bar{T}_{\text{smallest}}}{\sqrt{MS_{\text{error}}/n}}.$$

The Studentized range statistic can be used as part of a multiple comparisons procedure such as the Newman–Keuls procedure or Duncan's multiple range test (see Montgomery (1984) and Winer (1970)).

For a Studentized range statistic the probability integral, $P(q; v, r)$, for v degrees of freedom and r groups can be written as:

$$P(q; v, r) = C \int_0^\infty x^{v-1} e^{-vx^2/2} \left\{ r \int_{-\infty}^\infty \phi(y) [\Phi(y) - \Phi(y - qx)]^{r-1} dy \right\} dx,$$

where

$$C = \frac{v^{v/2}}{\Gamma(v/2) 2^{v/2-1}}, \quad \phi(y) = \frac{1}{\sqrt{2\pi}} e^{-y^2/2} \quad \text{and} \quad \Phi(y) = \int_{-\infty}^y \phi(t) dt.$$

The above two-dimensional integral is evaluated using nag_quad_2d_fin (d01da) with the upper and lower limits computed to give stated accuracy (see Section 7).

If the degrees of freedom v are greater than 2000 the probability integral can be approximated by its asymptotic form:

$$P(q; r) = r \int_{-\infty}^\infty \phi(y) [\Phi(y) - \Phi(y - q)]^{r-1} dy.$$

This integral is evaluated using nag_quad_1d_inf (d01am).

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Lund R E and Lund J R (1983) Algorithm AS 190: probabilities and upper quartiles for the studentized range *Appl. Statist.* **32(2)** 204–210

Montgomery D C (1984) *Design and Analysis of Experiments* Wiley

Winer B J (1970) *Statistical Principles in Experimental Design* McGraw–Hill

5 Parameters

5.1 Compulsory Input Parameters

1: **q** – REAL (KIND=nag_wp)

q , the Studentized range statistic.

Constraint: $q > 0.0$.

2: **v** – REAL (KIND=nag_wp)

v , the number of degrees of freedom for the experimental error.

Constraint: $v \geq 1.0$.

3: **ir** – INTEGER

r , the number of groups.

Constraint: $ir \geq 2$.

5.2 Optional Input Parameters

None.

5.3 Output Parameters

1: **result**

The result of the function.

2: **ifail** – INTEGER

ifail = 0 unless the function detects an error (see Section 5).

If on exit **ifail** = 1, then nag_stat_prob_studentized_range (g01em) returns to 0.0.

6 Error Indicators and Warnings

Note: nag_stat_prob_studentized_range (g01em) may return useful information for one or more of the following detected errors or warnings.

Errors or warnings detected by the function:

ifail = 1

On entry, $q \leq 0.0$,

or $v < 1.0$,

or $ir < 2$.

ifail = 2 (*warning*)

There is some doubt as to whether full accuracy has been achieved.

ifail = -99

An unexpected error has been triggered by this routine. Please contact NAG.

ifail = -399

Your licence key may have expired or may not have been installed correctly.

ifail = -999

Dynamic memory allocation failed.

7 Accuracy

The returned value will have absolute accuracy to at least four decimal places (usually five), unless **ifail** = 2. When **ifail** = 2 it is usual that the returned value will be a good estimate of the true value.

8 Further Comments

None.

9 Example

The lower tail probabilities for the distribution of the Studentized range statistic are computed and printed for a range of values of q , ν and r .

9.1 Program Text

```
function g01em_example

fprintf('g01em example results\n\n');

% Probability for Studentized range statistic distribution
q = [ 4.6543; 2.8099; 4.2636];
v = [10;      60.0;   5.0];
ir = [nag_int(5); 12;   4];

fprintf('  q      v      ir      probability\n');
for j = 1:numel(q);

    [p, ifail] = g01em( ...
                    q(j) , v(j),  ir(j));

    fprintf('%8.4f%6.1f%4d%12.4f\n', q(j), v(j), ir(j), p);
end
```

9.2 Program Results

g01em example results

q	v	ir	probability
4.6543	10.0	5	0.9500
2.8099	60.0	12	0.3000
4.2636	5.0	4	0.9000
