NAG Toolbox

nag_stat_test_shapiro_wilk (g01dd)

1 Purpose

nag_stat_test_shapiro_wilk (g01dd) calculates Shapiro and Wilk's W statistic and its significance level for testing Normality.

2 Syntax

```
[w, pw, ifail, a] = nag_stat_test_shapiro_wilk(x, 'n', n, 'a', a)
[w, pw, ifail, a] = g01dd(x, 'n', n, 'a', a)
```

Note: the interface to this routine has changed since earlier releases of the toolbox:

At Mark 23: **a** was made optional, **calwts** is no longer an input parameter; output parameters were reordered.

3 Description

nag_stat_test_shapiro_wilk (g01dd) calculates Shapiro and Wilk's W statistic and its significance level for any sample size between 3 and 5000. It is an adaptation of the Applied Statistics Algorithm AS R94, see Royston (1995). The full description of the theory behind this algorithm is given in Royston (1992).

Given a set of observations $x_1, x_2, \ldots, x_n$ sorted into either ascending or descending order (nag_sort_realvec_sort (m01ca) may be used to sort the data) this function calculates the value of Shapiro and Wilk's W statistic defined as:

$$W = \frac{\left(\sum_{i=1}^{n} a_i x_i\right)^2}{\sum_{i=1}^{n} (x_i - \bar{x})^2},$$

where $\bar{x} = \frac{1}{n}\sum_{1}^{n} x_i$ is the sample mean and a_i, for $i = 1, 2, \ldots, n$, are a set of 'weights' whose values depend only on the sample size n.

On exit, the values of a_i, for $i = 1, 2, \ldots, n$, are only of interest should you wish to call the function again to calculate **w** and its significance level for a different sample of the same size.

It is recommended that the function is used in conjunction with a Normal $(Q - Q)$ plot of the data. Functions nag_stat_normal_scores_exact (g01da) and nag_stat_normal_scores_approx (g01db) can be used to obtain the required Normal scores.

4 References

Royston J P (1982) Algorithm AS 181: the W test for normality *Appl. Statist.* **31** 176–180

Royston J P (1986) A remark on AS 181: the W test for normality *Appl. Statist.* **35** 232–234

Royston J P (1992) Approximating the Shapiro–Wilk's W test for non-normality *Statistics & Computing* **2** 117–119

Royston J P (1995) A remark on AS R94: A remark on Algorithm AS 181: the W test for normality *Appl. Statist.* **44(4)** 547–551

5 Parameters

5.1 Compulsory Input Parameters

1: **x**(**n**) – REAL (KIND=nag_wp) array

The ordered sample values, x_i, for $i = 1, 2, \ldots, n$.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the dimension of the arrays **x**, **a**. (An error is raised if these dimensions are not equal.)

n, the sample size.

Constraint: $3 \leq \mathbf{n} \leq 5000$.

2: **a**(**n**) – REAL (KIND=nag_wp) array

If supplied, **a** must contain the n weights as calculated in a previous call to nag_stat_test_shapiro_wilk (g01dd), otherwise **a** need not be set.

5.3 Output Parameters

1: **w** – REAL (KIND=nag_wp)

The value of the statistic, **w**.

2: **pw** – REAL (KIND=nag_wp)

The significance level of **w**.

3: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

4: **a**(**n**) – REAL (KIND=nag_wp) array

The n weights required to calculate **w**.

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, $\mathbf{n} < 3$.

ifail $= 2$

On entry, $\mathbf{n} > 5000$.

ifail $= 3$

On entry, the elements in **x** are not in ascending or descending order or are all equal.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

There may be a loss of significant figures for large n.

8 Further Comments

The time taken by nag_stat_test_shapiro_wilk (g01dd) depends roughly linearly on the value of n.

For very small samples the power of the test may not be very high.

The contents of the array **a** should not be modified between calls to nag_stat_test_shapiro_wilk (g01dd) for a given sample size, unless **calwts** is reset to *true* before each call of nag_stat_test_shapiro_wilk (g01dd).

The Shapiro and Wilk's W test is very sensitive to ties. If the data has been rounded the test can be improved by using Sheppard's correction to adjust the sum of squares about the mean. This produces an adjusted value of **w**,

$$WA = W\frac{\sum x_{(i)} - \bar{x}^2}{\left\{\sum_{i=1}^{n} x_{(i)} = \bar{x}^2 - \frac{n-1}{12}\omega^2\right\}},$$

where ω is the rounding width. WA can be compared with a standard Normal distribution, but a further approximation is given by Royston (1986).

If **n** > 5000, a value for **w** and **pw** is returned, but its accuracy may not be acceptable. See Section 4 for more details.

9 Example

This example tests the following two samples (each of size 20) for Normality.

Sample Number	Data
1	0.11, 7.87, 4.61, 10.14, 7.95, 3.14, 0.46, 4.43, 0.21, 4.75, 0.71, 1.52, 3.24, 0.93, 0.42, 4.97, 9.53, 4.55, 0.47, 6.66
2	1.36, 1.14, 2.92, 2.55, 1.46, 1.06, 5.27, −1.11, 3.48, 1.10, 0.88, −0.51, 1.46, 0.52, 6.20, 1.69, 0.08, 3.67, 2.81, 3.49

The elements of **a** are calculated only in the first call of nag_stat_test_shapiro_wilk (g01dd), and are re-used in the second call.

9.1 Program Text

```
function g01dd_example

fprintf('g01dd example results\n\n');

x(:,1) = [0.11;  0.21;  0.42;  0.46;  0.47;  0.71;  0.93;  1.52;  3.14;  3.24;
          4.43;  4.55;  4.61;  4.75;  4.97;  6.66;  7.87;  7.95;  9.53; 10.14];
x(:,2) = [1.36;  1.14;  2.92;  2.55;  1.46;  1.06;  5.27; -1.11;  3.48;  1.10;
          0.88; -0.51;  1.46;  0.52;  6.20;  1.69;  0.08;  3.67;  2.81;  3.49];

calwts = true;
n1 = nag_int(1);
% Loop over data sets
for j=1:size(x,2)

  % sort the data
```

```
  [x, ifail] = m01ca( ...
                      x, n1, 'Ascending');
  % Calculate test statistic (using weights 'a' in subsequent calls)
  if j==1
    [w, pw, ifail, a] = g01dd( ...
                               x(:,j));
  else
    [w, pw, ifail, a] = g01dd( ...
                               x(:,j), 'a', a);
  end

  % Display results
  fprintf('For sample number %d value of W statistic = %8.4f\n', j, w);
  fprintf('                      Significance level : %8.4f\n\n', pw);

end
```

9.2 Program Results

```
     g01dd example results

For sample number 1 value of W statistic =   0.9417
                    Significance level :   0.2579

For sample number 2 value of W statistic =   0.8857
                    Significance level :   0.0225
```