NAG Toolbox

nag_stat_prob_binomial (g01bj)

1 Purpose

nag_stat_prob_binomial (g01bj) returns the lower tail, upper tail and point probabilities associated with a binomial distribution.

2 Syntax

```
[plek, pgtk, peqk, ifail] = nag_stat_prob_binomial(n, p, k)
[plek, pgtk, peqk, ifail] = g01bj(n, p, k)
```

3 Description

Let X denote a random variable having a binomial distribution with parameters n and p ($n \geq 0$ and $0 < p < 1$). Then

$$\text{Prob}\{X = k\} = \binom{n}{k} p^k (1-p)^{n-k}, \quad k = 0, 1, \ldots, n.$$

The mean of the distribution is np and the variance is $np(1-p)$.

nag_stat_prob_binomial (g01bj) computes for given n, p and k the probabilities:

$$\begin{aligned} \textbf{plek} &= \text{Prob}\{X \leq k\} \\ \textbf{pgtk} &= \text{Prob}\{X > k\} \\ \textbf{peqk} &= \text{Prob}\{X = k\}. \end{aligned}$$

The method is similar to the method for the Poisson distribution described in KnÏsel (1986).

4 References

KnÏsel L (1986) Computation of the chi-square and Poisson distribution *SIAM J. Sci. Statist. Comput.* **7** 1022–1036

5 Parameters

5.1 Compulsory Input Parameters

1: **n** – INTEGER

The parameter n of the binomial distribution.

Constraint: $\mathbf{n} \geq 0$.

2: **p** – REAL (KIND=nag_wp)

The parameter p of the binomial distribution.

Constraint: $0.0 < \mathbf{p} < 1.0$.

3: **k** – INTEGER

The integer k which defines the required probabilities.

Constraint: $0 \leq \mathbf{k} \leq \mathbf{n}$.

5.2 Optional Input Parameters

None.

5.3 Output Parameters

1: **plek** – REAL (KIND=nag_wp)

The lower tail probability, Prob$\{X \leq k\}$.

2: **pgtk** – REAL (KIND=nag_wp)

The upper tail probability, Prob$\{X > k\}$.

3: **peqk** – REAL (KIND=nag_wp)

The point probability, Prob$\{X = k\}$.

4: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, **n** < 0.

ifail $= 2$

On entry, **p** ≤ 0.0,
or **p** ≥ 1.0.

ifail $= 3$

On entry, **k** < 0,
or **k** $>$ **n**.

ifail $= 4$

On entry, **n** is too large to be represented exactly as a double number.

ifail $= 5$

On entry, the variance ($= np(1-p)$) exceeds 10^6.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

Results are correct to a relative accuracy of at least 10^{-6} on machines with a precision of 9 or more decimal digits, and to a relative accuracy of at least 10^{-3} on machines of lower precision (provided that the results do not underflow to zero).

8 Further Comments

The time taken by nag_stat_prob_binomial (g01bj) depends on the variance ($= np(1-p)$) and on k. For given variance, the time is greatest when $k \approx np$ (= the mean), and is then approximately proportional to the square-root of the variance.

9 Example

This example reads values of n and p from a data file until end-of-file is reached, and prints the corresponding probabilities.

9.1 Program Text

```
     function g01bj_example

fprintf('g01bj example results\n\n');

n = nag_int([4    19     100    2000]);
k = nag_int([2    13     67     700]);
p =         [0.5  0.44   0.75   0.33];

fprintf('   n     p      k     plek      pgtk      peqk\n');
for i=1:4
  [plek, pgtk, peqk, ifail] = ...
  g01bj(n(i), p(i), k(i));
  fprintf('%5d%7.3f%5d%10.5f%10.5f%10.5f\n', n(i), p(i), k(i), ...
   plek, pgtk, peqk);
end
```

9.2 Program Results

```
     g01bj example results

   n     p      k     plek      pgtk      peqk
   4  0.500     2   0.68750   0.31250   0.37500
  19  0.440    13   0.99138   0.00862   0.01939
 100  0.750    67   0.04460   0.95540   0.01700
2000  0.330   700   0.97251   0.02749   0.00312
```