NAG Toolbox

nag_sparse_direct_real_gen_cond (f11mg)

1 Purpose

nag_sparse_direct_real_gen_cond (f11mg) computes an estimate of the reciprocal of the condition number of a sparse matrix given an LU factorization of the matrix computed by nag_sparse_direct_real_gen_lu (f11me).

2 Syntax

```
[rcond, ifail] = nag_sparse_direct_real_gen_cond(norm_p, n, il, lval, iu, uval, anorm)
[rcond, ifail] = f11mg(norm_p, n, il, lval, iu, uval, anorm)
```

3 Description

nag_sparse_direct_real_gen_cond (f11mg) estimates the condition number of a real sparse matrix A, in either the 1-norm or the ∞-norm:

$$\kappa_1(A) = \|A\|_1 \left\|A^{-1}\right\|_1 \quad \text{or} \quad \kappa_\infty(A) = \|A\|_\infty \left\|A^{-1}\right\|_\infty.$$

Note that $\kappa_\infty(A) = \kappa_1(A^{\mathrm{T}})$.

Because the condition number is infinite if A is singular, the function actually returns an estimate of the **reciprocal** of the condition number.

The function should be preceded by a call to nag_sparse_direct_real_gen_norm (f11ml) to compute $\|A\|_1$ or $\|A\|_\infty$, and a call to nag_sparse_direct_real_gen_lu (f11me) to compute the LU factorization of A. The function then estimates $\left\|A^{-1}\right\|_1$ or $\left\|A^{-1}\right\|_\infty$ and computes the reciprocal of the condition number.

4 References

None.

5 Parameters

5.1 Compulsory Input Parameters

1: **norm_p** – CHARACTER(1)

Indicates whether $\kappa_1(A)$ or $\kappa_\infty(A)$ is to be estimated.

norm_p = '1' or 'O'
$\kappa_1(A)$ is estimated.

norm_p = 'I'
$\kappa_\infty(A)$ is estimated.

Constraint: **norm_p** = '1', 'O' or 'I'.

2: **n** – INTEGER

n, the order of the matrix A.

Constraint: **n** ≥ 0.

3: **il**(:) – INTEGER array

The dimension of the array **il** must be at least as large as the dimension of the array of the same name in nag_sparse_direct_real_gen_lu (f11me)

Records the sparsity pattern of matrix L as computed by nag_sparse_direct_real_gen_lu (f11me).

4: **lval**(:) – REAL (KIND=nag_wp) array

The dimension of the array **lval** must be at least as large as the dimension of the array of the same name in nag_sparse_direct_real_gen_lu (f11me)

Records the nonzero values of matrix L and some nonzero values of matrix U as computed by nag_sparse_direct_real_gen_lu (f11me).

5: **iu**(:) – INTEGER array

The dimension of the array **iu** must be at least as large as the dimension of the array of the same name in nag_sparse_direct_real_gen_lu (f11me)

Records the sparsity pattern of matrix U as computed by nag_sparse_direct_real_gen_lu (f11me).

6: **uval**(:) – REAL (KIND=nag_wp) array

The dimension of the array **uval** must be at least as large as the dimension of the array of the same name in nag_sparse_direct_real_gen_lu (f11me)

Records some nonzero values of matrix U as computed by nag_sparse_direct_real_gen_lu (f11me).

7: **anorm** – REAL (KIND=nag_wp)

If **norm_p** = '1' or 'O', the 1-norm of the matrix A.

If **norm_p** = 'I', the ∞-norm of the matrix A.

anorm may be computed by calling nag_sparse_direct_real_gen_norm (f11ml) with the same value for the argument **norm_p**.

Constraint: **anorm** ≥ 0.0.

5.2 Optional Input Parameters

None.

5.3 Output Parameters

1: **rcond** – REAL (KIND=nag_wp)

An estimate of the reciprocal of the condition number of A. **rcond** is set to zero if exact singularity is detected or the estimate underflows. If **rcond** is less than ***machine precision***, A is singular to working precision.

2: **ifail** – INTEGER

ifail $= 0$ unless the function detects an error (see Section 5).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

Constraint: **anorm** ≥ 0.0.

Constraint: **n** ≥ 0.

On entry, **norm_p** $= \langle value \rangle$.
Constraint: **norm_p** = '1', 'O' or 'I'.

ifail $= -99$

An unexpected error has been triggered by this routine. Please contact NAG.

ifail $= -399$

Your licence key may have expired or may not have been installed correctly.

ifail $= -999$

Dynamic memory allocation failed.

7 Accuracy

The computed estimate **rcond** is never less than the true value ρ, and in practice is nearly always less than 10ρ, although examples can be constructed where **rcond** is much larger.

8 Further Comments

A call to nag_sparse_direct_real_gen_cond (f11mg) involves solving a number of systems of linear equations of the form $Ax = b$ or $A^{\mathrm{T}}x = b$.

9 Example

This example estimates the condition number in the 1-norm of the matrix A, where

$$A = \begin{pmatrix} 2.00 & 1.00 & 0 & 0 & 0 \\ 0 & 0 & 1.00 & -1.00 & 0 \\ 4.00 & 0 & 1.00 & 0 & 1.00 \\ 0 & 0 & 0 & 1.00 & 2.00 \\ 0 & -2.00 & 0 & 0 & 3.00 \end{pmatrix}.$$

Here A is nonsymmetric and must first be factorized by nag_sparse_direct_real_gen_lu (f11me). The true condition number in the 1-norm is 20.25.

9.1 Program Text

```
function f11mg_example

fprintf('f11mg example results\n\n');

% Sparse matrix A
n      = nag_int(5);
a      = [        2;  4; 1; -2; 1;  1; -1; 1; 1; 2; 3];
icolzp = [nag_int(1); 3; 5;  7; 9; 12];
irowix = [nag_int(1); 3; 1;  5; 2;  3;  2; 4; 3; 4; 5];
nz     = icolzp(n+1) - 1;

% Calculate COLAMD permutation
spec   = 'M';
iprm   = zeros(7*n, 1, nag_int_name);

[iprm, ifail] = f11md( ...
                       spec, n, icolzp, irowix, iprm);

% Factorise
thresh = 1;
nzlmx  = nag_int(8*nz);
nzlumx = nag_int(8*nz);
nzumx  = nag_int(8*nz);
```

```
[iprm, nzlumx, il, lval, iu, uval, nnzl, nnzu, flop, ifail] = ...
  f11me( ...
         n, irowix, a, iprm, thresh, nzlmx, nzlumx, nzumx);

% Calculate Norm
norm_p = '1';
[anorm, ifail] = f11ml( ...
                        norm_p, n, icolzp, irowix, a);

% Calculate condition number
[rcond, ifail] = f11mg( ...
                        norm_p, n, il, lval, iu, uval, anorm);

fprintf('Norm             = %7.3f\n', anorm);
fprintf('Condition number = %7.3f\n', 1/rcond);
```

9.2 Program Results

```
     f11mg example results

Norm             =   6.000
Condition number =  20.250
```