NAG Toolbox

nag_lapack_zggglm (f08zp)

1 Purpose

nag_lapack_zggglm (f08zp) solves a complex general Gauss–Markov linear (least squares) model problem.

2 Syntax

```
[a, b, d, x, y, info] = nag_lapack_zggglm(a, b, d, 'm', m, 'n', n, 'p', p)
[a, b, d, x, y, info] = f08zp(a, b, d, 'm', m, 'n', n, 'p', p)
```

3 Description

nag_lapack_zggglm (f08zp) solves the complex general Gauss–Markov linear model (GLM) problem

$$\underset{x}{\text{minimize}} \|y\|_2 \quad \text{subject to} \quad d = Ax + By$$

where A is an m by n matrix, B is an m by p matrix and d is an m element vector. It is assumed that $n \leq m \leq n + p$, $\text{rank}(A) = n$ and $\text{rank}(E) = m$, where $E = \begin{pmatrix} A & B \end{pmatrix}$. Under these assumptions, the problem has a unique solution x and a minimal 2-norm solution y, which is obtained using a generalized QR factorization of the matrices A and B.

In particular, if the matrix B is square and nonsingular, then the GLM problem is equivalent to the weighted linear least squares problem

$$\underset{x}{\text{minimize}} \left\|B^{-1}(d - Ax)\right\|_2.$$

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia

Anderson E, Bai Z and Dongarra J (1992) Generalized *QR* factorization and its applications *Linear Algebra Appl. (Volume 162–164)* 243–271

5 Parameters

5.1 Compulsory Input Parameters

1: **a**$(lda, :)$ – COMPLEX (KIND=nag_wp) array

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$.

The second dimension of the array **a** must be at least $\max(1, \mathbf{n})$.

The m by n matrix A.

2: **b**$(ldb, :)$ – COMPLEX (KIND=nag_wp) array

The first dimension of the array **b** must be at least $\max(1, \mathbf{m})$.

The second dimension of the array **b** must be at least $\max(1, \mathbf{p})$.

The m by p matrix B.

3: **d**($\mathbf{m}$) – COMPLEX (KIND=nag_wp) array

The left-hand side vector d of the GLM equation.

5.2 Optional Input Parameters

1: **m** – INTEGER

Default: the dimension of the array **d** and the first dimension of the array **b**. (An error is raised if these dimensions are not equal.)

m, the number of rows of the matrices A and B.

Constraint: $\mathbf{m} \geq 0$.

2: **n** – INTEGER

Default: the second dimension of the array **a**.

n, the number of columns of the matrix A.

Constraint: $0 \leq \mathbf{n} \leq \mathbf{m}$.

3: **p** – INTEGER

Default: the second dimension of the array **b**.

p, the number of columns of the matrix B.

Constraint: $\mathbf{p} \geq \mathbf{m} - \mathbf{n}$.

5.3 Output Parameters

1: **a**($lda, :$) – COMPLEX (KIND=nag_wp) array

The first dimension of the array **a** will be $\max(1, \mathbf{m})$.

The second dimension of the array **a** will be $\max(1, \mathbf{n})$.

2: **b**($ldb, :$) – COMPLEX (KIND=nag_wp) array

The first dimension of the array **b** will be $\max(1, \mathbf{m})$.

The second dimension of the array **b** will be $\max(1, \mathbf{p})$.

3: **d**($\mathbf{m}$) – COMPLEX (KIND=nag_wp) array

4: **x**($\mathbf{n}$) – COMPLEX (KIND=nag_wp) array

The solution vector x of the GLM problem.

5: **y**($\mathbf{p}$) – COMPLEX (KIND=nag_wp) array

The solution vector y of the GLM problem.

6: **info** – INTEGER

$\mathbf{info} = 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

$\mathbf{info} = -i$

If $\mathbf{info} = -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **m**, 2: **n**, 3: **p**, 4: **a**, 5: **lda**, 6: **b**, 7: **ldb**, 8: **d**, 9: **x**, 10: **y**, 11: **work**, 12: **lwork**, 13: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info $= 1$

The upper triangular factor R associated with A in the generalized RQ factorization of the pair (A, B) is singular, so that $\mathrm{rank}(A) < m$; the least squares solution could not be computed.

info $= 2$

The bottom $(N - M)$ by $(N - M)$ part of the upper trapezoidal factor T associated with B in the generalized QR factorization of the pair (A, B) is singular, so that $\mathrm{rank}\begin{pmatrix} A & B \end{pmatrix} < N$; the least squares solutions could not be computed.

7 Accuracy

For an error analysis, see Anderson *et al.* (1992). See also Section 4.6 of Anderson *et al.* (1999).

8 Further Comments

When $p = m \geq n$, the total number of real floating-point operations is approximately $\frac{8}{3}(2m^3 - n^3) + 16nm^2$; when $p = m = n$, the total number of real floating-point operations is approximately $\frac{56}{3}m^3$.

9 Example

This example solves the weighted least squares problem

$$\underset{x}{\mathrm{minimize}} \left\| B^{-1}(d - Ax) \right\|_2,$$

where

$$B = \begin{pmatrix} 0.5 - 1.0i & & & \\ & 1.0 - 2.0i & & \\ & & 2.0 - 3.0i & \\ & & & 5.0 - 4.0i \end{pmatrix},$$

$$d = \begin{pmatrix} 6.00 - 0.40i \\ -5.27 + 0.90i \\ 2.72 - 2.13i \\ -1.30 - 2.80i \end{pmatrix}$$

and

$$A = \begin{pmatrix} 0.96 - 0.81i & -0.03 + 0.96i & -0.91 + 2.06i \\ -0.98 + 1.98i & -1.20 + 0.19i & -0.66 + 0.42i \\ 0.62 - 0.46i & 1.01 + 0.02i & 0.63 - 0.17i \\ 1.08 - 0.28i & 0.20 - 0.12i & -0.07 + 1.23i \end{pmatrix}.$$

Note that the block size (NB) of 64 assumed in this example is not realistic for such a small problem, but should be suitable for large problems.

9.1 Program Text

```
      function f08zp_example

fprintf('f08zp example results\n\n');

a = [ 0.96 - 0.81i, -0.03 + 0.96i, -0.91 + 2.06i;
     -0.98 + 1.98i, -1.20 + 0.19i, -0.66 + 0.42i;
      0.62 - 0.46i,  1.01 + 0.02i,  0.63 - 0.17i;
```

```
      1.08 - 0.28i,  0.20 - 0.12i, -0.07 + 1.23i];
b = [ 0.5 - 1i,  0 + 0i,  0 + 0i,  0 + 0i;
      0   + 0i,  1 - 2i,  0 + 0i,  0 + 0i;
      0   + 0i,  0 + 0i,  2 - 3i,  0 + 0i;
      0   + 0i,  0 + 0i,  0 + 0i,  5 - 4i];
d = [ 6.00 - 0.40i;
     -5.27 + 0.90i;
      2.72 - 2.13i;
     -1.30 - 2.80i];

%Solve complex general Guass-Markov linear model
[~, ~, ~, x, y, info] = f08zp( ...
                               a, b, d);

disp('Weighted least-squares solution');
disp(x);
ncols  = nag_int(80);
indent = nag_int(0);
[ifail] = x04db( ...
                 'Gen', ' ', y, 'B', '1P,E10.2', 'Residual vector', ...
                 'N', 'N', ncols, indent);
sqres = norm(y,2);
fprintf('\nSquare root of the residual sum of squares\n%11.2e\n', ...
        sqres);
```

9.2 Program Results

```
     f08zp example results

Weighted least-squares solution
  -0.9846 + 1.9950i
   3.9929 - 4.9748i
  -3.0026 + 0.9994i

 Residual vector
   (  1.26E-04, -4.66E-04)
   (  1.11E-03, -8.61E-04)
   (  3.84E-03, -1.82E-03)
   (  2.03E-03,  3.02E-03)

Square root of the residual sum of squares
   5.79e-03
```