NAG Toolbox

nag_lapack_dggglm (f08zb)

1 Purpose

nag_lapack_dggglm (f08zb) solves a real general Gauss–Markov linear (least squares) model problem.

2 Syntax

```
[a, b, d, x, y, info] = nag_lapack_dggglm(a, b, d, 'm', m, 'n', n, 'p', p)
[a, b, d, x, y, info] = f08zb(a, b, d, 'm', m, 'n', n, 'p', p)
```

3 Description

nag_lapack_dggglm (f08zb) solves the real general Gauss–Markov linear model (GLM) problem

$$\underset{x}{\text{minimize}}\|y\|_2 \quad \text{subject to} \quad d = Ax + By$$

where A is an m by n matrix, B is an m by p matrix and d is an m element vector. It is assumed that $n \leq m \leq n + p$, $\text{rank}(A) = n$ and $\text{rank}(E) = m$, where $E = \begin{pmatrix} A & B \end{pmatrix}$. Under these assumptions, the problem has a unique solution x and a minimal 2-norm solution y, which is obtained using a generalized QR factorization of the matrices A and B.

In particular, if the matrix B is square and nonsingular, then the GLM problem is equivalent to the weighted linear least squares problem

$$\underset{x}{\text{minimize}}\left\|B^{-1}(d - Ax)\right\|_2.$$

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia

Anderson E, Bai Z and Dongarra J (1992) Generalized *QR* factorization and its applications *Linear Algebra Appl. (Volume 162–164)* 243–271

5 Parameters

5.1 Compulsory Input Parameters

1: **a**(lda, :) – REAL (KIND=nag_wp) array

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$.

The second dimension of the array **a** must be at least $\max(1, \mathbf{n})$.

The m by n matrix A.

2: **b**(ldb, :) – REAL (KIND=nag_wp) array

The first dimension of the array **b** must be at least $\max(1, \mathbf{m})$.

The second dimension of the array **b** must be at least $\max(1, \mathbf{p})$.

The m by p matrix B.

3: **d**(**m**) – REAL (KIND=nag_wp) array

The left-hand side vector d of the GLM equation.

5.2 Optional Input Parameters

1: **m** – INTEGER

Default: the dimension of the array **d** and the first dimension of the array **b**. (An error is raised if these dimensions are not equal.)

m, the number of rows of the matrices A and B.

Constraint: $\mathbf{m} \geq 0$.

2: **n** – INTEGER

Default: the second dimension of the array **a**.

n, the number of columns of the matrix A.

Constraint: $0 \leq \mathbf{n} \leq \mathbf{m}$.

3: **p** – INTEGER

Default: the second dimension of the array **b**.

p, the number of columns of the matrix B.

Constraint: $\mathbf{p} \geq \mathbf{m} - \mathbf{n}$.

5.3 Output Parameters

1: **a**(lda, :) – REAL (KIND=nag_wp) array

The first dimension of the array **a** will be $\max(1, \mathbf{m})$.

The second dimension of the array **a** will be $\max(1, \mathbf{n})$.

2: **b**(ldb, :) – REAL (KIND=nag_wp) array

The first dimension of the array **b** will be $\max(1, \mathbf{m})$.

The second dimension of the array **b** will be $\max(1, \mathbf{p})$.

3: **d**(**m**) – REAL (KIND=nag_wp) array

4: **x**(**n**) – REAL (KIND=nag_wp) array

The solution vector x of the GLM problem.

5: **y**(**p**) – REAL (KIND=nag_wp) array

The solution vector y of the GLM problem.

6: **info** – INTEGER

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **m**, 2: **n**, 3: **p**, 4: **a**, 5: **lda**, 6: **b**, 7: **ldb**, 8: **d**, 9: **x**, 10: **y**, 11: **work**, 12: **lwork**, 13: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info $= 1$

The upper triangular factor R associated with A in the generalized RQ factorization of the pair (A, B) is singular, so that $\mathrm{rank}(A) < m$; the least squares solution could not be computed.

info $= 2$

The bottom $(N - M)$ by $(N - M)$ part of the upper trapezoidal factor T associated with B in the generalized QR factorization of the pair (A, B) is singular, so that $\mathrm{rank}\begin{pmatrix} A & B \end{pmatrix} < N$; the least squares solutions could not be computed.

7 Accuracy

For an error analysis, see Anderson *et al.* (1992). See also Section 4.6 of Anderson *et al.* (1999).

8 Further Comments

When $p = m \geq n$, the total number of floating-point operations is approximately $\frac{2}{3}\left(2m^3 - n^3\right) + 4nm^2$; when $p = m = n$, the total number of floating-point operations is approximately $\frac{14}{3}m^3$.

9 Example

This example solves the weighted least squares problem

$$\underset{x}{\mathrm{minimize}}\left\|B^{-1}(d - Ax)\right\|_2,$$

where

$$B = \begin{pmatrix} 0.5 & 0.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 2.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 5.0 \end{pmatrix}, \quad d = \begin{pmatrix} 1.32 \\ -4.00 \\ 5.52 \\ 3.24 \end{pmatrix} \quad \text{and} \quad A = \begin{pmatrix} -0.57 & -1.28 & -0.39 \\ -1.93 & 1.08 & -0.31 \\ 2.30 & 0.24 & -0.40 \\ -0.02 & 1.03 & -1.43 \end{pmatrix}.$$

9.1 Program Text

```
function f08zb_example

fprintf('f08zb example results\n\n');

% Minimize ||y|| given Ax + By = d
a = [-0.57, -1.28, -0.39;
     -1.93,  1.08, -0.31;
      2.30,  0.24, -0.40;
     -0.02,  1.03, -1.43];
b = [ 0.5, 0, 0, 0;
      0,   1, 0, 0;
      0,   0, 2, 0;
      0,   0, 0, 5];
d = [ 1.32;
     -4.00;
      5.52;
      3.24];

[~, ~, ~, x, y, info] = f08zb( ...
                              a, b, d);

disp('Weighted least-squares solution');
fprintf('%12.4f',x);
fprintf('\n');
```

```
fprintf('Residual vector\n  ');
fprintf('%12.2e',y);
sqres = norm(y,2);
fprintf('\n\nSquare root of the residual sum of squares\n  %12.2e\n', ...
        sqres);
```

9.2 Program Results

```
     f08zb example results

Weighted least-squares solution
      1.9889     -1.0058     -2.9911
Residual vector
     -6.37e-04   -2.45e-03   -4.72e-03    7.70e-03

Square root of the residual sum of squares
      9.38e-03
```