NAG Toolbox

nag_lapack_dpbstf (f08uf)

1 Purpose

nag_lapack_dpbstf (f08uf) computes a split Cholesky factorization of a real symmetric positive definite band matrix.

2 Syntax

```
[bb, info] = nag_lapack_dpbstf(uplo, kb, bb, 'n', n)
[bb, info] = f08uf(uplo, kb, bb, 'n', n)
```

3 Description

nag_lapack_dpbstf (f08uf) computes a split Cholesky factorization of a real symmetric positive definite band matrix B. It is designed to be used in conjunction with nag_lapack_dsbgst (f08ue).

The factorization has the form $B = S^{\mathrm{T}}S$, where S is a band matrix of the same bandwidth as B and the following structure: S is upper triangular in the first $(n+k)/2$ rows, and transposed — hence, lower triangular — in the remaining rows. For example, if $n = 9$ and $k = 2$, then

$$S = \begin{pmatrix} s_{11} & s_{12} & s_{13} & & & & & & \\ & s_{22} & s_{23} & s_{24} & & & & & \\ & & s_{33} & s_{34} & s_{35} & & & & \\ & & & s_{44} & s_{45} & & & & \\ & & & & s_{55} & & & & \\ & & & s_{64} & s_{65} & s_{66} & & & \\ & & & & s_{75} & s_{76} & s_{77} & & \\ & & & & & s_{86} & s_{87} & s_{88} & \\ & & & & & & s_{97} & s_{98} & s_{99} \end{pmatrix}.$$

4 References

None.

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – CHARACTER(1)

Indicates whether the upper or lower triangular part of B is stored.

uplo = 'U'
The upper triangular part of B is stored.

uplo = 'L'
The lower triangular part of B is stored.

Constraint: **uplo** = 'U' or 'L'.

2: **kb** – INTEGER

If **uplo** = 'U', the number of superdiagonals, k_b, of the matrix B.

If **uplo** = 'L', the number of subdiagonals, k_b, of the matrix B.

Constraint: **kb** ≥ 0.

3: **bb**($ldbb$, :) – REAL (KIND=nag_wp) array

The first dimension of the array **bb** must be at least **kb** $+ 1$.

The second dimension of the array **bb** must be at least $\max(1, \mathbf{n})$.

The n by n symmetric positive definite band matrix B.

The matrix is stored in rows 1 to $k_b + 1$, more precisely,

if **uplo** = 'U', the elements of the upper triangle of B within the band must be stored with element B_{ij} in $\mathbf{bb}(k_b + 1 + i - j, j)$ for $\max(1, j - k_b) \leq i \leq j$;

if **uplo** = 'L', the elements of the lower triangle of B within the band must be stored with element B_{ij} in $\mathbf{bb}(1 + i - j, j)$ for $j \leq i \leq \min(n, j + k_b)$.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the first dimension of the array **bb** and the second dimension of the array **bb**. (An error is raised if these dimensions are not equal.)

n, the order of the matrix B.

Constraint: $\mathbf{n} \geq 0$.

5.3 Output Parameters

1: **bb**($ldbb$, :) – REAL (KIND=nag_wp) array

The first dimension of the array **bb** will be **kb** $+ 1$.

The second dimension of the array **bb** will be $\max(1, \mathbf{n})$.

B stores the elements of its split Cholesky factor S.

2: **info** – INTEGER

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **n**, 3: **kb**, 4: **bb**, 5: **ldbb**, 6: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

If **info** $= i$, the factorization could not be completed, because the updated element $b(i, i)$ would be the square root of a negative number. Hence B is not positive definite. This may indicate an error in forming the matrix B.

7 Accuracy

The computed factor S is the exact factor of a perturbed matrix $(B+E)$, where

$$|E| \leq c(k+1)\epsilon|S^{\mathrm{T}}||S|,$$

$c(k+1)$ is a modest linear function of $k+1$, and ϵ is the ***machine precision***. It follows that $|e_{ij}| \leq c(k+1)\epsilon\sqrt{(b_{ii}b_{jj})}$.

8 Further Comments

The total number of floating-point operations is approximately $n(k+1)^2$, assuming $n \gg k$.

A call to nag_lapack_dpbstf (f08uf) may be followed by a call to nag_lapack_dsbgst (f08ue) to solve the generalized eigenproblem $Az = \lambda Bz$, where A and B are banded and B is positive definite.

The complex analogue of this function is nag_lapack_zpbstf (f08ut).

9 Example

See Section 10 in nag_lapack_dsbgst (f08ue).

9.1 Program Text

```
     function f08uf_example

fprintf('f08uf example results\n\n');

% Sove Az = lambda Bz
% A and B are the symmetric banded positive definite matrices:
n = 4;
% A has 2 off-diagonals
ka = nag_int(2);
a = [ 0.24,  0.39,  0.42,  0.00;
      0.39, -0.11,  0.79,  0.63;
      0.42,  0.79, -0.25,  0.48;
      0.00,  0.63,  0.48, -0.03];
% B has 1 off-diagonal
kb = nag_int(1);
b = [ 2.07   0.95   0.00   0.00;
      0.95   1.69  -0.29   0.00;
      0.00  -0.29   0.65  -0.33;
      0.00   0.00  -0.33   1.17];

% Convert to general banded format ...
[~, ab, ifail] = f01zc( ...
 'P', ka, ka, a, zeros(ka+ka+1,n));
[~, bb, ifail] = f01zc( ...
 'P', kb, kb, b, zeros(kb+kb+1,n));
% ... and chop to give 'Upper' symmetric banded format
ab = ab(1:ka+1,1:n);
bb = bb(1:kb+1,1:n);

% Factorize B
uplo = 'Upper';
[ub, info] = f08uf( ...
        uplo, kb, bb);

% Reduce problem to standard form Cy = lambda*y
vect = 'N';
[cb, x, info] = f08ue( ...
        vect, uplo, ka, kb, ab, ub);

% Find eigenvalues lambda
jobz = 'No Vectors';
```

```
[~, w, ~, info] = f08ha( ...
  jobz, uplo, ka, cb);

disp('Eigenvalues:');
disp(w');
```

9.2 Program Results

```
     f08uf example results

Eigenvalues:
   -0.8305   -0.6401    0.0992    1.8525
```