NAG Toolbox

nag_lapack_ztrsen (f08qu)

1 Purpose

nag_lapack_ztrsen (f08qu) reorders the Schur factorization of a complex general matrix so that a selected cluster of eigenvalues appears in the leading elements on the diagonal of the Schur form. The function also optionally computes the reciprocal condition numbers of the cluster of eigenvalues and/or the invariant subspace.

2 Syntax

```
[t, q, w, m, s, sep, info] = nag_lapack_ztrsen(job, compq, select, t, q, 'n', n)
[t, q, w, m, s, sep, info] = f08qu(job, compq, select, t, q, 'n', n)
```

3 Description

nag_lapack_ztrsen (f08qu) reorders the Schur factorization of a complex general matrix $A = QTQ^{\mathrm{H}}$, so that a selected cluster of eigenvalues appears in the leading diagonal elements of the Schur form.

The reordered Schur form $\tilde{T}$ is computed by a unitary similarity transformation: $\tilde{T} = Z^{\mathrm{H}}TZ$. Optionally the updated matrix $\tilde{Q}$ of Schur vectors is computed as $\tilde{Q} = QZ$, giving $A = \tilde{Q}\tilde{T}\tilde{Q}^{\mathrm{H}}$.

Let $\tilde{T} = \begin{pmatrix} T_{11} & T_{12} \\ 0 & T_{22} \end{pmatrix}$, where the selected eigenvalues are precisely the eigenvalues of the leading m by m sub-matrix T_{11}. Let $\tilde{Q}$ be correspondingly partitioned as $\begin{pmatrix} Q_1 & Q_2 \end{pmatrix}$ where Q_1 consists of the first m columns of Q. Then $AQ_1 = Q_1T_{11}$, and so the m columns of Q_1 form an orthonormal basis for the invariant subspace corresponding to the selected cluster of eigenvalues.

Optionally the function also computes estimates of the reciprocal condition numbers of the average of the cluster of eigenvalues and of the invariant subspace.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **job** – CHARACTER(1)

Indicates whether condition numbers are required for the cluster of eigenvalues and/or the invariant subspace.

job = 'N'
No condition numbers are required.

job = 'E'
Only the condition number for the cluster of eigenvalues is computed.

job = 'V'
Only the condition number for the invariant subspace is computed.

job = 'B'
Condition numbers for both the cluster of eigenvalues and the invariant subspace are computed.

Constraint: **job** = 'N', 'E', 'V' or 'B'.

2: **compq** – CHARACTER(1)

Indicates whether the matrix Q of Schur vectors is to be updated.

compq = 'V'
The matrix Q of Schur vectors is updated.

compq = 'N'
No Schur vectors are updated.

Constraint: **compq** = 'V' or 'N'.

3: **select**(:) – LOGICAL array

The dimension of the array **select** must be at least $\max(1, \mathbf{n})$

Specifies the eigenvalues in the selected cluster. To select a complex eigenvalue λ_j, **select**(j) must be set *true*.

4: **t**(ldt, :) – COMPLEX (KIND=nag_wp) array

The first dimension of the array **t** must be at least $\max(1, \mathbf{n})$.

The second dimension of the array **t** must be at least $\max(1, \mathbf{n})$.

The n by n upper triangular matrix T, as returned by nag_lapack_zhseqr (f08ps).

5: **q**(ldq, :) – COMPLEX (KIND=nag_wp) array

The first dimension, ldq, of the array **q** must satisfy

if **compq** = 'V', $ldq \geq \max(1, \mathbf{n})$;
if **compq** = 'N', $ldq \geq 1$.

The second dimension of the array **q** must be at least $\max(1, \mathbf{n})$ if **compq** = 'V' and at least 1 if **compq** = 'N'.

If **compq** = 'V', **q** must contain the n by n unitary matrix Q of Schur vectors, as returned by nag_lapack_zhseqr (f08ps).

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the first dimension of the array **t** and the second dimension of the array **t**. (An error is raised if these dimensions are not equal.)

n, the order of the matrix T.

Constraint: $\mathbf{n} \geq 0$.

5.3 Output Parameters

1: **t**(ldt, :) – COMPLEX (KIND=nag_wp) array

The first dimension of the array **t** will be $\max(1, \mathbf{n})$.

The second dimension of the array **t** will be $\max(1, \mathbf{n})$.

t stores the updated matrix $\tilde{T}$.

2: **q**$(ldq,:)$ – COMPLEX (KIND=nag_wp) array

The first dimension, ldq, of the array **q** will be

if **compq** $=$ 'V', $ldq = \max(1, \mathbf{n})$;
if **compq** $=$ 'N', $ldq = 1$.

The second dimension of the array **q** will be $\max(1, \mathbf{n})$ if **compq** $=$ 'V' and at least 1 if **compq** $=$ 'N'.

If **compq** $=$ 'V', **q** contains the updated matrix of Schur vectors; the first m columns of Q form an orthonormal basis for the specified invariant subspace.

If **compq** $=$ 'N', **q** is not referenced.

3: **w**$(:)$ – COMPLEX (KIND=nag_wp) array

The dimension of the array **w** will be $\max(1, \mathbf{n})$

The reordered eigenvalues of $\tilde{T}$. The eigenvalues are stored in the same order as on the diagonal of $\tilde{T}$.

4: **m** – INTEGER

m, the dimension of the specified invariant subspace, which is the same as the number of selected eigenvalues (see **select**); $0 \leq m \leq n$.

5: **s** – REAL (KIND=nag_wp)

If **job** $=$ 'E' or 'B', **s** is a lower bound on the reciprocal condition number of the average of the selected cluster of eigenvalues. If $\mathbf{m} = 0$ or $\mathbf{n}$, $\mathbf{s} = 1$.

If **job** $=$ 'N' or 'V', **s** is not referenced.

6: **sep** – REAL (KIND=nag_wp)

If **job** $=$ 'V' or 'B', **sep** is the estimated reciprocal condition number of the specified invariant subspace. If $\mathbf{m} = 0$ or $\mathbf{n}$, $\mathbf{sep} = \|T\|$.

If **job** $=$ 'N' or 'E', **sep** is not referenced.

7: **info** – INTEGER

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **job**, 2: **compq**, 3: **select**, 4: **n**, 5: **t**, 6: **ldt**, 7: **q**, 8: **ldq**, 9: **w**, 10: **m**, 11: **s**, 12: **sep**, 13: **work**, 14: **lwork**, 15: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The computed matrix $\tilde{T}$ is similar to a matrix $(T + E)$, where

$$\|E\|_2 = O(\epsilon)\|T\|_2,$$

and ϵ is the ***machine precision***.

s cannot underestimate the true reciprocal condition number by more than a factor of $\sqrt{\min(m, n-m)}$. **sep** may differ from the true value by $\sqrt{m(n-m)}$. The angle between the computed invariant subspace and the true subspace is $\frac{O(\epsilon)\|A\|_2}{sep}$.

The values of the eigenvalues are never changed by the reordering.

8 Further Comments

The real analogue of this function is nag_lapack_dtrsen (f08qg).

9 Example

This example reorders the Schur factorization of the matrix $A = QTQ^{\mathrm{H}}$ such that the eigenvalues stored in elements t_{11} and t_{44} appear as the leading elements on the diagonal of the reordered matrix $\tilde{T}$, where

$$T = \begin{pmatrix} -6.0004 - 6.9999i & 0.3637 - 0.3656i & -0.1880 + 0.4787i & 0.8785 - 0.2539i \\ 0.0000 + 0.0000i & -5.0000 + 2.0060i & -0.0307 - 0.7217i & -0.2290 + 0.1313i \\ 0.0000 + 0.0000i & 0.0000 + 0.0000i & 7.9982 - 0.9964i & 0.9357 + 0.5359i \\ 0.0000 + 0.0000i & 0.0000 + 0.0000i & 0.0000 + 0.0000i & 3.0023 - 3.9998i \end{pmatrix}$$

and

$$Q = \begin{pmatrix} -0.8347 - 0.1364i & -0.0628 + 0.3806i & 0.2765 - 0.0846i & 0.0633 - 0.2199i \\ 0.0664 - 0.2968i & 0.2365 + 0.5240i & -0.5877 - 0.4208i & 0.0835 + 0.2183i \\ -0.0362 - 0.3215i & 0.3143 - 0.5473i & 0.0576 - 0.5736i & 0.0057 - 0.4058i \\ 0.0086 + 0.2958i & -0.3416 - 0.0757i & -0.1900 - 0.1600i & 0.8327 - 0.1868i \end{pmatrix}.$$

The original matrix A is given in Section 10 in nag_lapack_zunghr (f08nt).

9.1 Program Text

```
function f08qu_example

fprintf('f08qu example results\n\n');

ilo = nag_int(1);
ihi = nag_int(4);
a = [ -3.97 - 5.04i, -4.11 + 3.70i, -0.34 + 1.01i,  1.29 - 0.86i;
       0.34 - 1.50i,  1.52 - 0.43i,  1.88 - 5.38i,  3.36 + 0.65i;
       3.31 - 3.85i,  2.50 + 3.45i,  0.88 - 1.08i,  0.64 - 1.48i;
      -1.10 + 0.82i,  1.81 - 1.59i,  3.25 + 1.33i,  1.57 - 3.44i];

% Reduce A to upper Hessenberg Form
[H, tau, info] = f08ns( ...
 ilo, ihi, a);

% Form Q
[Q, info] = f08nt( ...
    ilo, ihi, H, tau);

% Schur factorize H = Y*T*Y' and form Z = QY  A = QY*T*(QQY)'
job   = 'Schur form';
compz = 'Vectors';
[T, w, Z, info] = f08ps( ...
  job, compz, ilo, ihi, H, Q);

job = 'Both';
compq = 'Vectors';
select = [true; false;  false;  true];
[T, Q, w, m, s, sep, info] = f08qu( ...
     job, compq, select, T, Z);
```

```
disp('selected eigenvalues:');
disp(w(1:m));
fprintf('Condition number estimate for selected eigenvalues = %7.4f\n',1/s);
fprintf('Condition number estimate for specified subspace   = %7.4f\n',1/sep);
```

9.2 Program Results

```
     f08qu example results

selected eigenvalues:
  -6.0004 - 6.9998i
   3.0023 - 3.9998i

Condition number estimate for selected eigenvalues =  1.0196
Condition number estimate for specified subspace   =  0.1822
```