NAG Toolbox

nag_lapack_dtrexc (f08qf)

1 Purpose

nag_lapack_dtrexc (f08qf) reorders the Schur factorization of a real general matrix.

2 Syntax

```
[t, q, ifst, ilst, info] = nag_lapack_dtrexc(compq, t, q, ifst, ilst, 'n', n)
[t, q, ifst, ilst, info] = f08qf(compq, t, q, ifst, ilst, 'n', n)
```

3 Description

nag_lapack_dtrexc (f08qf) reorders the Schur factorization of a real general matrix $A = QTQ^{\mathrm{T}}$, so that the diagonal element or block of T with row index **ifst** is moved to row **ilst**.

The reordered Schur form $\tilde{T}$ is computed by an orthogonal similarity transformation: $\tilde{T} = Z^{\mathrm{T}}TZ$. Optionally the updated matrix $\tilde{Q}$ of Schur vectors is computed as $\tilde{Q} = QZ$, giving $A = \tilde{Q}\tilde{T}\tilde{Q}^{\mathrm{T}}$.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **compq** – CHARACTER(1)

Indicates whether the matrix Q of Schur vectors is to be updated.

compq = 'V'
The matrix Q of Schur vectors is updated.

compq = 'N'
No Schur vectors are updated.

Constraint: **compq** = 'V' or 'N'.

2: **t**$(ldt, :)$ – REAL (KIND=nag_wp) array

The first dimension of the array **t** must be at least $\max(1, \mathbf{n})$.

The second dimension of the array **t** must be at least $\max(1, \mathbf{n})$.

The n by n upper quasi-triangular matrix T in canonical Schur form, as returned by nag_lapack_dhseqr (f08pe).

3: **q**$(ldq, :)$ – REAL (KIND=nag_wp) array

The first dimension, ldq, of the array **q** must satisfy

if **compq** = 'V', $ldq \geq \max(1, \mathbf{n})$;
if **compq** = 'N', $ldq \geq 1$.

The second dimension of the array **q** must be at least $\max(1, \mathbf{n})$ if **compq** = 'V' and at least 1 if **compq** = 'N'.

If **compq** = 'V', **q** must contain the n by n orthogonal matrix Q of Schur vectors.

4: **ifst** – INTEGER
5: **ilst** – INTEGER

ifst and **ilst** must specify the reordering of the diagonal elements or blocks of T. The element or block with row index **ifst** is moved to row **ilst** by a sequence of exchanges between adjacent elements or blocks.

Constraint: $1 \leq$ **ifst** $\leq$ **n** and $1 \leq$ **ilst** $\leq$ **n**.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the first dimension of the array **t** and the second dimension of the array **t**. (An error is raised if these dimensions are not equal.)

n, the order of the matrix T.

Constraint: **n** ≥ 0.

5.3 Output Parameters

1: **t**$(ldt,:)$ – REAL (KIND=nag_wp) array

The first dimension of the array **t** will be $\max(1,\mathbf{n})$.

The second dimension of the array **t** will be $\max(1,\mathbf{n})$.

t stores the updated matrix $\tilde{T}$. See also Section 9.

2: **q**$(ldq,:)$ – REAL (KIND=nag_wp) array

The first dimension, ldq, of the array **q** will be

if **compq** = 'V', $ldq = \max(1,\mathbf{n})$;
if **compq** = 'N', $ldq = 1$.

The second dimension of the array **q** will be $\max(1,\mathbf{n})$ if **compq** = 'V' and at least 1 if **compq** = 'N'.

If **compq** = 'V', **q** contains the updated matrix of Schur vectors.

If **compq** = 'N', **q** is not referenced.

3: **ifst** – INTEGER
4: **ilst** – INTEGER

If **ifst** pointed to the second row of a 2 by 2 block on entry, it is changed to point to the first row. **ilst** always points to the first row of the block in its final position (which may differ from its input value by ± 1).

5: **info** – INTEGER

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **compq**, 2: **n**, 3: **t**, 4: **ldt**, 5: **q**, 6: **ldq**, 7: **ifst**, 8: **ilst**, 9: **work**, 10: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info $= 1$ (*warning*)

Two adjacent diagonal elements or blocks could not be successfully exchanged. This error can only occur if the exchange involves at least one 2 by 2 block; it implies that the problem is very ill-conditioned, and that the eigenvalues of the two blocks are very close. On exit, T may have been partially reordered, and **ilst** points to the first row of the current position of the block being moved; Q (if requested) is updated consistently with T.

7 Accuracy

The computed matrix $\tilde{T}$ is exactly similar to a matrix $(T + E)$, where

$$\|E\|_2 = O(\epsilon)\|T\|_2,$$

and ϵ is the ***machine precision***.

Note that if a 2 by 2 diagonal block is involved in the reordering, its off-diagonal elements are in general changed; the diagonal elements and the eigenvalues of the block are unchanged unless the block is sufficiently ill-conditioned, in which case they may be noticeably altered. It is possible for a 2 by 2 block to break into two 1 by 1 blocks, i.e., for a pair of complex eigenvalues to become purely real. The values of real eigenvalues however are never changed by the reordering.

8 Further Comments

The total number of floating-point operations is approximately $6nr$ if **compq** $=$ 'N', and $12nr$ if **compq** $=$ 'V', where $r = |\mathbf{ifst} - \mathbf{ilst}|$.

The input matrix T must be in canonical Schur form, as is the output matrix $\tilde{T}$. This has the following structure.

If all the computed eigenvalues are real, T is upper triangular and its diagonal elements are the eigenvalues.

If some of the computed eigenvalues form complex conjugate pairs, then T has 2 by 2 diagonal blocks. Each diagonal block has the form

$$\begin{pmatrix} t_{ii} & t_{i,i+1} \\ t_{i+1,i} & t_{i+1,i+1} \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \alpha \end{pmatrix}$$

where $\beta\gamma < 0$. The corresponding eigenvalues are $\alpha \pm \sqrt{\beta\gamma}$.

The complex analogue of this function is nag_lapack_ztrexc (f08qt).

9 Example

This example reorders the Schur factorization of the matrix T so that the 2 by 2 block with row index 2 is moved to row 1, where

$$T = \begin{pmatrix} 0.80 & -0.11 & 0.01 & 0.03 \\ 0.00 & -0.10 & 0.25 & 0.35 \\ 0.00 & -0.65 & -0.10 & 0.20 \\ 0.00 & 0.00 & 0.00 & -0.10 \end{pmatrix}.$$

9.1 Program Text

```
     function f08qf_example

fprintf('f08qf example results\n\n');

% Block triangular matrix T from Schur factorization
t = [0.8, -0.11,  0.01,  0.03;
     0,   -0.11,  0.25,  0.35;
     0,   -0.65, -0.10,  0.20;
     0,    0,     0,    -0.10];

% Reorder T to move 2-by-2 block with index 2 to row 1
compq = 'No update';
q = [0];
ifst = nag_int(2);
ilst = nag_int(1);
[t, q, ifst, ilst, info] = f08qf( ...
   compq, t, q, ifst, ilst);

disp('Reordered Schur Form');
disp(t);
```

9.2 Program Results

```
     f08qf example results

Reordered Schur Form
   -0.1050   -0.6465    0.0877    0.2054
    0.2513   -0.1050    0.0919    0.3480
         0         0    0.8000   -0.0113
         0         0         0   -0.1000
```