NAG Toolbox

nag_lapack_zgeesx (f08pp)

1 Purpose

nag_lapack_zgeesx (f08pp) computes the eigenvalues, the Schur form T, and, optionally, the matrix of Schur vectors Z for an n by n complex nonsymmetric matrix A.

2 Syntax

```
[a, sdim, w, vs, rconde, rcondv, info] = nag_lapack_zgeesx(jobvs, sort, select,
sense, a, 'n', n)
```

```
[a, sdim, w, vs, rconde, rcondv, info] = f08pp(jobvs, sort, select, sense, a,
'n', n)
```

3 Description

The Schur factorization of A is given by

$$A = ZTZ^{\mathrm{H}},$$

where Z, the matrix of Schur vectors, is unitary and T is the Schur form. A complex matrix is in Schur form if it is upper triangular.

Optionally, nag_lapack_zgeesx (f08pp) also orders the eigenvalues on the diagonal of the Schur form so that selected eigenvalues are at the top left; computes a reciprocal condition number for the average of the selected eigenvalues (**rconde**); and computes a reciprocal condition number for the right invariant subspace corresponding to the selected eigenvalues (**rcondv**). The leading columns of Z form an orthonormal basis for this invariant subspace.

For further explanation of the reciprocal condition numbers **rconde** and **rcondv**, see Section 4.8 of Anderson *et al.* (1999) (where these quantities are called s and sep respectively).

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **jobvs** – CHARACTER(1)

If **jobvs** = 'N', Schur vectors are not computed.

If **jobvs** = 'V', Schur vectors are computed.

Constraint: **jobvs** = 'N' or 'V'.

2: **sort** – CHARACTER(1)

Specifies whether or not to order the eigenvalues on the diagonal of the Schur form.

sort = 'N'
Eigenvalues are not ordered.

sort = 'S'
Eigenvalues are ordered (see **select**).

Constraint: **sort** = 'N' or 'S'.

3: **select** – LOGICAL FUNCTION, supplied by the user.

If **sort** = 'S', **select** is used to select eigenvalues to sort to the top left of the Schur form.

If **sort** = 'N', **select** is not referenced and nag_lapack_zgeesx (f08pp) may be called with the string `'f08pnz'`.

An eigenvalue $\mathbf{w}(j)$ is selected if $\mathbf{select}(\mathbf{w}(j))$ is *true*.

```
[result] = select(w)
```

Input Parameters

1: **w** – COMPLEX (KIND=nag_wp)

The real and imaginary parts of the eigenvalue.

Output Parameters

1: **result**

$result = true$ for selected eigenvalues.

4: **sense** – CHARACTER(1)

Determines which reciprocal condition numbers are computed.

sense = 'N'
None are computed.

sense = 'E'
Computed for average of selected eigenvalues only.

sense = 'V'
Computed for selected right invariant subspace only.

sense = 'B'
Computed for both.

If **sense** = 'E', 'V' or 'B', **sort** = 'S'.

Constraint: **sense** = 'N', 'E', 'V' or 'B'.

5: $\mathbf{a}(lda,:)$ – COMPLEX (KIND=nag_wp) array

The first dimension of the array **a** must be at least $\max(1,\mathbf{n})$.

The second dimension of the array **a** must be at least $\max(1,\mathbf{n})$.

The n by n matrix A.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the first dimension of the array **a** and the second dimension of the array **a**. (An error is raised if these dimensions are not equal.)

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5.3 Output Parameters

1: **a**$(lda,:)$ – COMPLEX (KIND=nag_wp) array

The first dimension of the array **a** will be $\max(1,\mathbf{n})$.

The second dimension of the array **a** will be $\max(1,\mathbf{n})$.

a stores its Schur form T.

2: **sdim** – INTEGER

If **sort** = 'N', **sdim** $= 0$.

If **sort** = 'S', **sdim** = number of eigenvalues for which **select** is *true*.

3: **w**$(:)$ – COMPLEX (KIND=nag_wp) array

The dimension of the array **w** will be $\max(1,\mathbf{n})$

Contains the computed eigenvalues, in the same order that they appear on the diagonal of the output Schur form T.

4: **vs**$(ldvs,:)$ – COMPLEX (KIND=nag_wp) array

The first dimension, $ldvs$, of the array **vs** will be

if **jobvs** = 'V', $ldvs = \max(1,\mathbf{n})$;
otherwise $ldvs = 1$.

The second dimension of the array **vs** will be $\max(1,\mathbf{n})$ if **jobvs** = 'V' and 1 otherwise.

If **jobvs** = 'V', **vs** contains the unitary matrix Z of Schur vectors.

If **jobvs** = 'N', **vs** is not referenced.

5: **rconde** – REAL (KIND=nag_wp)

If **sense** = 'E' or 'B', contains the reciprocal condition number for the average of the selected eigenvalues.

If **sense** = 'N' or 'V', **rconde** is not referenced.

6: **rcondv** – REAL (KIND=nag_wp)

If **sense** = 'V' or 'B', **rcondv** contains the reciprocal condition number for the selected right invariant subspace.

If **sense** = 'N' or 'E', **rcondv** is not referenced.

7: **info** – INTEGER

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **jobvs**, 2: **sort**, 3: **select**, 4: **sense**, 5: **n**, 6: **a**, 7: **lda**, 8: **sdim**, 9: **w**, 10: **vs**, 11: **ldvs**, 12: **rconde**, 13: **rcondv**, 14: **work**, 15: **lwork**, 16: **rwork**, 17: **bwork**, 18: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info $= 1$ to **n**

If **info** $= i$ and $i \leq$ **n**, the QR algorithm failed to compute all the eigenvalues.

info $=$ **n** $+ 1$ (*warning*)

The eigenvalues could not be reordered because some eigenvalues were too close to separate (the problem is very ill-conditioned).

info $=$ **n** $+ 2$ (*warning*)

After reordering, roundoff changed values of some complex eigenvalues so that leading eigenvalues in the Schur form no longer satisfy **select** $= \mathit{true}$. This could also be caused by underflow due to scaling.

7 Accuracy

The computed Schur factorization satisfies

$$A + E = ZTZ^{\mathrm{H}},$$

where

$$\|E\|_2 = O(\epsilon)\|A\|_2,$$

and ϵ is the ***machine precision***. See Section 4.8 of Anderson *et al.* (1999) for further details.

8 Further Comments

The total number of floating-point operations is proportional to n^3.

The real analogue of this function is nag_lapack_dgeesx (f08pb).

9 Example

This example finds the Schur factorization of the matrix

$$A = \begin{pmatrix} -3.97 - 5.04i & -4.11 + 3.70i & -0.34 + 1.01i & 1.29 - 0.86i \\ 0.34 - 1.50i & 1.52 - 0.43i & 1.88 - 5.38i & 3.36 + 0.65i \\ 3.31 - 3.85i & 2.50 + 3.45i & 0.88 - 1.08i & 0.64 - 1.48i \\ -1.10 + 0.82i & 1.81 - 1.59i & 3.25 + 1.33i & 1.57 - 3.44i \end{pmatrix},$$

such that the eigenvalues of A with positive real part of are the top left diagonal elements of the Schur form, T. Estimates of the condition numbers for the selected eigenvalue cluster and corresponding invariant subspace are also returned.

Note that the block size (NB) of 64 assumed in this example is not realistic for such a small problem, but should be suitable for large problems.

9.1 Program Text

```
function f08pp_example

fprintf('f08pp example results\n\n');

% Complex matrix A
a = [ -3.97 - 5.04i, -4.11 + 3.70i, -0.34 + 1.01i,  1.29 - 0.86i;
       0.34 - 1.50i,  1.52 - 0.43i,  1.88 - 5.38i,  3.36 + 0.65i;
       3.31 - 3.85i,  2.50 + 3.45i,  0.88 - 1.08i,  0.64 - 1.48i;
      -1.10 + 0.82i,  1.81 - 1.59i,  3.25 + 1.33i,  1.57 - 3.44i];

% Schur vectors of A, selecting eigenvalues with positive real parts
jobvs = 'Vectors (Schur)';
sortp = 'Sort';
select = @(w) (real(w) > 0 );
sense = 'Both reciprocal condition numbers';
[~, sdim, w, V, rconde, rcondv, info] = ...
f08pp( ...
       jobvs, sortp, select, sense, a);

fprintf('Number of selected eigenvalues = %2d\n',sdim);

disp('Selected eigenvalues:');
disp(w(1:sdim));
disp('Corresponding eigenvectors:');
% Normalize eigenvectors before printing
disp(V(:,1:sdim)/diag(V(1,1:sdim)));

fprintf('Projection norm  for the selected eigenvalues = %7.4f\n',1/rconde);
fprintf('Condition number for the selected eigenvalues = %7.4f\n\n',1/rcondv);

anorm = norm(a);
erbde = x02aj*anorm/rconde;
erbdv = x02aj*anorm/rcondv;
fprintf('%-61s = %9.1e\n', ...
        'Approximate asymptotic error bound for selected eigenvalues', erbde);
fprintf('%-61s = %9.1e\n', ...
        'Approximate asymptotic error bound for the invariant subspace', ...
        erbdv);
```

9.2 Program Results

```
f08pp example results

Number of selected eigenvalues =  2
Selected eigenvalues:
   7.9982 - 0.9964i
   3.0023 - 3.9998i

Corresponding eigenvectors:
   1.0000 + 0.0000i   1.0000 + 0.0000i
  -1.1841 - 1.8270i  -0.8886 + 0.3089i
   0.7402 - 1.7252i   2.1298 - 0.0489i
  -0.4668 - 0.6356i   0.6784 + 4.4559i

Projection norm  for the selected eigenvalues =  1.0072
Condition number for the selected eigenvalues =  0.1081

Approximate asymptotic error bound for selected eigenvalues   =   1.0e-15
Approximate asymptotic error bound for the invariant subspace =   1.1e-16
```