NAG Toolbox

nag_lapack_zgees (f08pn)

1 Purpose

nag_lapack_zgees (f08pn) computes the eigenvalues, the Schur form T, and, optionally, the matrix of Schur vectors Z for an n by n complex nonsymmetric matrix A.

2 Syntax

```
[a, sdim, w, vs, info] = nag_lapack_zgees(jobvs, sort, select, a, 'n', n)
[a, sdim, w, vs, info] = f08pn(jobvs, sort, select, a, 'n', n)
```

3 Description

The Schur factorization of A is given by

$$A = ZTZ^{\mathrm{H}},$$

where Z, the matrix of Schur vectors, is unitary and T is the Schur form. A complex matrix is in Schur form if it is upper triangular.

Optionally, nag_lapack_zgees (f08pn) also orders the eigenvalues on the diagonal of the Schur form so that selected eigenvalues are at the top left. The leading columns of Z form an orthonormal basis for the invariant subspace corresponding to the selected eigenvalues.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **jobvs** – CHARACTER(1)

If **jobvs** = 'N', Schur vectors are not computed.

If **jobvs** = 'V', Schur vectors are computed.

Constraint: **jobvs** = 'N' or 'V'.

2: **sort** – CHARACTER(1)

Specifies whether or not to order the eigenvalues on the diagonal of the Schur form.

sort = 'N'
Eigenvalues are not ordered.

sort = 'S'
Eigenvalues are ordered (see **select**).

Constraint: **sort** = 'N' or 'S'.

3: **select** – LOGICAL FUNCTION, supplied by the user.

If **sort** = 'S', **select** is used to select eigenvalues to sort to the top left of the Schur form.

If **sort** = 'N', **select** is not referenced and nag_lapack_zgees (f08pn) may be called with the string `'f08pnz'`.

An eigenvalue $\mathbf{w}(j)$ is selected if **select**$(\mathbf{w}(j))$ is *true*.

```
[result] = select(w)
```

Input Parameters

1: **w** – COMPLEX (KIND=nag_wp)

The real and imaginary parts of the eigenvalue.

Output Parameters

1: **result**

$result = true$ for selected eigenvalues.

4: **a**$(lda, :)$ – COMPLEX (KIND=nag_wp) array

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$.

The second dimension of the array **a** must be at least $\max(1, \mathbf{n})$.

The n by n matrix A.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the first dimension of the array **a** and the second dimension of the array **a**. (An error is raised if these dimensions are not equal.)

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5.3 Output Parameters

1: **a**$(lda, :)$ – COMPLEX (KIND=nag_wp) array

The first dimension of the array **a** will be $\max(1, \mathbf{n})$.

The second dimension of the array **a** will be $\max(1, \mathbf{n})$.

a stores its Schur form T.

2: **sdim** – INTEGER

If **sort** = 'N', **sdim** = 0.

If **sort** = 'S', **sdim** = number of eigenvalues for which **select** is *true*.

3: **w**$(:)$ – COMPLEX (KIND=nag_wp) array

The dimension of the array **w** will be $\max(1, \mathbf{n})$

Contains the computed eigenvalues, in the same order that they appear on the diagonal of the output Schur form T.

4: **vs**$(ldvs,:)$ – COMPLEX (KIND=nag_wp) array

The first dimension, $ldvs$, of the array **vs** will be

if **jobvs** = 'V', $ldvs = \max(1, \mathbf{n})$;
otherwise $ldvs = 1$.

The second dimension of the array **vs** will be $\max(1, \mathbf{n})$ if **jobvs** = 'V' and 1 otherwise.

If **jobvs** = 'V', **vs** contains the unitary matrix Z of Schur vectors.

If **jobvs** = 'N', **vs** is not referenced.

5: **info** – INTEGER

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **jobvs**, 2: **sort**, 3: **select**, 4: **n**, 5: **a**, 6: **lda**, 7: **sdim**, 8: **w**, 9: **vs**, 10: **ldvs**, 11: **work**, 12: **lwork**, 13: **rwork**, 14: **bwork**, 15: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info $= 1$ to **n**

If **info** $= i$ and $i \leq \mathbf{n}$, the QR algorithm failed to compute all the eigenvalues.

info $= \mathbf{n} + 1$ (*warning*)

The eigenvalues could not be reordered because some eigenvalues were too close to separate (the problem is very ill-conditioned).

info $= \mathbf{n} + 2$ (*warning*)

After reordering, roundoff changed values of some complex eigenvalues so that leading eigenvalues in the Schur form no longer satisfy **select** $= true$. This could also be caused by underflow due to scaling.

7 Accuracy

The computed Schur factorization satisfies

$$A + E = ZTZ^{\mathrm{H}},$$

where

$$\|E\|_2 = O(\epsilon)\|A\|_2,$$

and ϵ is the ***machine precision***. See Section 4.8 of Anderson *et al.* (1999) for further details.

8 Further Comments

The total number of floating-point operations is proportional to n^3.

The real analogue of this function is nag_lapack_dgees (f08pa).

9 Example

This example finds the Schur factorization of the matrix

$$A = \begin{pmatrix} -3.97 - 5.04i & -4.11 + 3.70i & -0.34 + 1.01i & 1.29 - 0.86i \\ 0.34 - 1.50i & 1.52 - 0.43i & 1.88 - 5.38i & 3.36 + 0.65i \\ 3.31 - 3.85i & 2.50 + 3.45i & 0.88 - 1.08i & 0.64 - 1.48i \\ -1.10 + 0.82i & 1.81 - 1.59i & 3.25 + 1.33i & 1.57 - 3.44i \end{pmatrix}.$$

Note that the block size (NB) of 64 assumed in this example is not realistic for such a small problem, but should be suitable for large problems.

9.1 Program Text

```
function f08pn_example

fprintf('f08pn example results\n\n');

% Complex matrix A
a = [-3.97 - 5.04i, -4.11 + 3.70i, -0.34 + 1.01i,  1.29 - 0.86i;
      0.34 - 1.50i,  1.52 - 0.43i,  1.88 - 5.38i,  3.36 + 0.65i;
      3.31 - 3.85i,  2.50 + 3.45i,  0.88 - 1.08i,  0.64 - 1.48i;
     -1.10 + 0.82i,  1.81 - 1.59i,  3.25 + 1.33i,  1.57 - 3.44i];

% Schur vectors of A, selecting all eigenvalues
jobvs = 'Vectors (Schur)';
sortp = 'No sort';
[~, sdim, w, ~, info] = f08pn( ...
                              jobvs, sortp, @select, a);

disp('Eigenvalues');
disp(w);
```

9.2 Program Results

```
f08pn example results

Eigenvalues
  -6.0004 - 6.9998i
  -5.0000 + 2.0060i
   7.9982 - 0.9964i
   3.0023 - 3.9998i
```