NAG Toolbox

nag_lapack_zgehrd (f08ns)

1 Purpose

nag_lapack_zgehrd (f08ns) reduces a complex general matrix to Hessenberg form.

2 Syntax

```
[a, tau, info] = nag_lapack_zgehrd(ilo, ihi, a, 'n', n)
[a, tau, info] = f08ns(ilo, ihi, a, 'n', n)
```

3 Description

nag_lapack_zgehrd (f08ns) reduces a complex general matrix A to upper Hessenberg form H by a unitary similarity transformation: $A = QHQ^{\mathrm{H}}$. H has real subdiagonal elements.

The matrix Q is not formed explicitly, but is represented as a product of elementary reflectors (see the F08 Chapter Introduction for details). Functions are provided to work with Q in this representation (see Section 9).

The function can take advantage of a previous call to nag_lapack_zgebal (f08nv), which may produce a matrix with the structure:

$$\begin{pmatrix} A_{11} & A_{12} & A_{13} \\ & A_{22} & A_{23} \\ & & A_{33} \end{pmatrix}$$

where A_{11} and A_{33} are upper triangular. If so, only the central diagonal block A_{22}, in rows and columns i_{lo} to i_{hi}, needs to be reduced to Hessenberg form (the blocks A_{12} and A_{23} will also be affected by the reduction). Therefore the values of i_{lo} and i_{hi} determined by nag_lapack_zgebal (f08nv) can be supplied to the function directly. If nag_lapack_zgebal (f08nv) has not previously been called however, then i_{lo} must be set to 1 and i_{hi} to n.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **ilo** – INTEGER

2: **ihi** – INTEGER

If A has been output by nag_lapack_zgebal (f08nv), then **ilo** and **ihi** **must** contain the values returned by that function. Otherwise, **ilo** must be set to 1 and **ihi** to **n**.

Constraints:

if $\mathbf{n} > 0$, $1 \le \mathbf{ilo} \le \mathbf{ihi} \le \mathbf{n}$;
if $\mathbf{n} = 0$, $\mathbf{ilo} = 1$ and $\mathbf{ihi} = 0$.

3: **a**(lda, :) – COMPLEX (KIND=nag_wp) array

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$.

The second dimension of the array **a** must be at least $\max(1, \mathbf{n})$.

The n by n general matrix A.

5.2 Optional Input Parameters

1: **n** – INTEGER

Default: the first dimension of the array **a** and the second dimension of the array **a**.

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5.3 Output Parameters

1: **a**$(lda, :)$ – COMPLEX (KIND=nag_wp) array

The first dimension of the array **a** will be $\max(1, \mathbf{n})$.

The second dimension of the array **a** will be $\max(1, \mathbf{n})$.

a stores the upper Hessenberg matrix H and details of the unitary matrix Q. The subdiagonal elements of H are real.

2: **tau**$(:)$ – COMPLEX (KIND=nag_wp) array

The dimension of the array **tau** will be $\max(1, \mathbf{n} - 1)$

Further details of the unitary matrix Q.

3: **info** – INTEGER

$\mathbf{info} = 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

$\mathbf{info} = -i$

If $\mathbf{info} = -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **n**, 2: **ilo**, 3: **ihi**, 4: **a**, 5: **lda**, 6: **tau**, 7: **work**, 8: **lwork**, 9: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The computed Hessenberg matrix H is exactly similar to a nearby matrix $(A + E)$, where

$$\|E\|_2 \leq c(n)\epsilon\|A\|_2,$$

$c(n)$ is a modestly increasing function of n, and ϵ is the ***machine precision***.

The elements of H themselves may be sensitive to small perturbations in A or to rounding errors in the computation, but this does not affect the stability of the eigenvalues, eigenvectors or Schur factorization.

8 Further Comments

The total number of real floating-point operations is approximately $\frac{8}{3}q^2(2q + 3n)$, where $q = i_{\mathrm{hi}} - i_{\mathrm{lo}}$; if $i_{\mathrm{lo}} = 1$ and $i_{\mathrm{hi}} = n$, the number is approximately $\frac{40}{3}n^3$.

To form the unitary matrix Q nag_lapack_zgehrd (f08ns) may be followed by a call to nag_lapack_zunghr (f08nt):

```
[a, info] = f08nt(ilo, ihi, a, tau);
```

To apply Q to an m by n complex matrix C nag_lapack_zgehrd (f08ns) may be followed by a call to nag_lapack_zunmhr (f08nu). For example,

```
[c, info] = f08nu('Left', 'No Transpose', ilo, ihi, a, tau, c);
```

forms the matrix product QC.

The real analogue of this function is nag_lapack_dgehrd (f08ne).

9 Example

This example computes the upper Hessenberg form of the matrix A, where

$$A = \begin{pmatrix} -3.97-5.04i & -4.11+3.70i & -0.34+1.01i & 1.29-0.86i \\ 0.34-1.50i & 1.52-0.43i & 1.88-5.38i & 3.36+0.65i \\ 3.31-3.85i & 2.50+3.45i & 0.88-1.08i & 0.64-1.48i \\ -1.10+0.82i & 1.81-1.59i & 3.25+1.33i & 1.57-3.44i \end{pmatrix}.$$

9.1 Program Text

```
function f08ns_example

fprintf('f08ns example results\n\n');

ilo = nag_int(1);
ihi = nag_int(4);
a = [ -3.97 - 5.04i, -4.11 + 3.70i, -0.34 + 1.01i,  1.29 - 0.86i;
       0.34 - 1.50i,  1.52 - 0.43i,  1.88 - 5.38i,  3.36 + 0.65i;
       3.31 - 3.85i,  2.50 + 3.45i,  0.88 - 1.08i,  0.64 - 1.48i;
      -1.10 + 0.82i,  1.81 - 1.59i,  3.25 + 1.33i,  1.57 - 3.44i];

% Reduce A to upper Hessenberg Form
[H, tau, info] = f08ns(ilo, ihi, a);

disp('Upper Hessenberg Form H');
disp(H);
```

9.2 Program Results

```
f08ns example results

Upper Hessenberg Form H
  -3.9700 - 5.0400i  -1.1318 - 2.5693i  -4.6027 - 0.1426i  -1.4249 + 1.7330i
  -5.4797 + 0.0000i   1.8585 - 1.5502i   4.4145 - 0.7638i  -0.4805 - 1.1976i
   0.6932 - 0.4829i   6.2673 + 0.0000i  -0.4504 - 0.0290i  -1.3467 + 1.6579i
  -0.2113 + 0.0864i   0.1242 - 0.2289i  -3.5000 + 0.0000i   2.5619 - 3.3708i
```