NAG Toolbox

nag_lapack_zgesdd (f08kr)

1 Purpose

nag_lapack_zgesdd (f08kr) computes the singular value decomposition (SVD) of a complex m by n matrix A, optionally computing the left and/or right singular vectors, by using a divide-and-conquer method.

2 Syntax

```
[a, s, u, vt, info] = nag_lapack_zgesdd(jobz, a, 'm', m, 'n', n)
[a, s, u, vt, info] = f08kr(jobz, a, 'm', m, 'n', n)
```

3 Description

The SVD is written as

$$A = U\Sigma V^{\mathrm{H}},$$

where Σ is an m by n matrix which is zero except for its $\min(m,n)$ diagonal elements, U is an m by m unitary matrix, and V is an n by n unitary matrix. The diagonal elements of Σ are the singular values of A; they are real and non-negative, and are returned in descending order. The first $\min(m,n)$ columns of U and V are the left and right singular vectors of A.

Note that the function returns V^{H}, not V.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **jobz** – CHARACTER(1)

Specifies options for computing all or part of the matrix U.

jobz = 'A'
All m columns of U and all n rows of V^{H} are returned in the arrays **u** and **vt**.

jobz = 'S'
The first $\min(m,n)$ columns of U and the first $\min(m,n)$ rows of V^{H} are returned in the arrays **u** and **vt**.

jobz = 'O'
If **m** $\geq$ **n**, the first n columns of U are overwritten on the array **a** and all rows of V^{H} are returned in the array **vt**. Otherwise, all columns of U are returned in the array **u** and the first m rows of V^{H} are overwritten in the array **vt**.

jobz = 'N'
No columns of U or rows of V^{H} are computed.

Constraint: **jobz** = 'A', 'S', 'O' or 'N'.

2: **a**$(lda,:)$ – COMPLEX (KIND=nag_wp) array

The first dimension of the array **a** must be at least $\max(1,\mathbf{m})$.

The second dimension of the array **a** must be at least $\max(1,\mathbf{n})$.

The m by n matrix A.

5.2 Optional Input Parameters

1: **m** – INTEGER

Default: the first dimension of the array **a**.

m, the number of rows of the matrix A.

Constraint: $\mathbf{m} \geq 0$.

2: **n** – INTEGER

Default: the second dimension of the array **a**.

n, the number of columns of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5.3 Output Parameters

1: **a**$(lda,:)$ – COMPLEX (KIND=nag_wp) array

The first dimension of the array **a** will be $\max(1,\mathbf{m})$.

The second dimension of the array **a** will be $\max(1,\mathbf{n})$.

If **jobz** = 'O', **a** is overwritten with the first n columns of U (the left singular vectors, stored column-wise) if $\mathbf{m} \geq \mathbf{n}$; **a** is overwritten with the first m rows of V^{H} (the right singular vectors, stored row-wise) otherwise.

If **jobz** $\neq$ 'O', the contents of **a** are destroyed.

2: **s**$(\min(\mathbf{m},\mathbf{n}))$ – REAL (KIND=nag_wp) array

The singular values of A, sorted so that $\mathbf{s}(i) \geq \mathbf{s}(i+1)$.

3: **u**$(ldu,:)$ – COMPLEX (KIND=nag_wp) array

The first dimension, ldu, of the array **u** will be

if **jobz** = 'S' or 'A' or **jobz** = 'O' and $\mathbf{m} < \mathbf{n}$, $ldu = \max(1,\mathbf{m})$;
otherwise $ldu = 1$.

The second dimension of the array **u** will be $\max(1,\mathbf{m})$ if **jobz** = 'A' or **jobz** = 'O' and $\mathbf{m} < \mathbf{n}$, $\max(1,\min(\mathbf{m},\mathbf{n}))$ if **jobz** = 'S' and 1 otherwise.

If **jobz** = 'A' or **jobz** = 'O' and $\mathbf{m} < \mathbf{n}$, **u** contains the m by m unitary matrix U.

If **jobz** = 'S', **u** contains the first $\min(m,n)$ columns of U (the left singular vectors, stored column-wise).

If **jobz** = 'O' and $\mathbf{m} \geq \mathbf{n}$, or **jobz** = 'N', **u** is not referenced.

4: **vt**$(ldvt,:)$ – COMPLEX (KIND=nag_wp) array

The first dimension, $ldvt$, of the array **vt** will be

if **jobz** = 'A' or **jobz** = 'O' and **m** $\geq$ **n**, $ldvt = \max(1, \mathbf{n})$;
if **jobz** = 'S', $ldvt = \max(1, \min(\mathbf{m}, \mathbf{n}))$;
otherwise $ldvt = 1$.

The second dimension of the array **vt** will be $\max(1, \mathbf{n})$ if **jobz** = 'A' or 'S' or **jobz** = 'O' and **m** $\geq$ **n** and 1 otherwise.

If **jobz** = 'A' or **jobz** = 'O' and **m** $\geq$ **n**, **vt** contains the n by n unitary matrix V^{H}.

If **jobz** = 'S', **vt** contains the first $\min(m, n)$ rows of V^{H} (the right singular vectors, stored row-wise).

If **jobz** = 'O' and **m** $<$ **n**, or **jobz** = 'N', **vt** is not referenced.

5: **info** – INTEGER

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **jobz**, 2: **m**, 3: **n**, 4: **a**, 5: **lda**, 6: **s**, 7: **u**, 8: **ldu**, 9: **vt**, 10: **ldvt**, 11: **work**, 12: **lwork**, 13: **rwork**, 14: **iwork**, 15: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

nag_lapack_zgesdd (f08kr) did not converge, the updating process failed.

7 Accuracy

The computed singular value decomposition is nearly the exact singular value decomposition for a nearby matrix $(A + E)$, where

$$\|E\|_2 = O(\epsilon)\|A\|_2,$$

and ϵ is the ***machine precision***. In addition, the computed singular vectors are nearly orthogonal to working precision. See Section 4.9 of Anderson *et al.* (1999) for further details.

8 Further Comments

The total number of floating-point operations is approximately proportional to mn^2 when $m > n$ and m^2n otherwise.

The singular values are returned in descending order.

The real analogue of this function is nag_lapack_dgesdd (f08kd).

9 Example

This example finds the singular values and left and right singular vectors of the 4 by 6 matrix

$$A = \begin{pmatrix} 0.96+0.81i & -0.98-1.98i & 0.62+0.46i & -0.37-0.38i & 0.83-0.51i & 1.08+0.28i \\ -0.03-0.96i & -1.20-0.19i & 1.01-0.02i & 0.19+0.54i & 0.20-0.01i & 0.20+0.12i \\ -0.91-2.06i & -0.66-0.42i & 0.63+0.17i & -0.98+0.36i & -0.17+0.46i & -0.07-1.23i \\ -0.05-0.41i & -0.81-0.56i & -1.11-0.60i & 0.22+0.20i & 1.47-1.59i & 0.26-0.26i \end{pmatrix},$$

together with approximate error bounds for the computed singular values and vectors.

The example program for nag_lapack_zgesvd (f08kp) illustrates finding a singular value decomposition for the case $m \geq n$.

9.1 Program Text

```
function f08kr_example

fprintf('f08kr example results\n\n');

% SVD of complex matrix A
a = [ 0.96 + 0.81i, -0.98 - 1.98i,  0.62 + 0.46i, ...
                    -0.37 - 0.38i,  0.83 - 0.51i,  1.08 + 0.28i;
     -0.03 - 0.96i, -1.20 - 0.19i,  1.01 - 0.02i, ...
                     0.19 + 0.54i,  0.20 - 0.01i,  0.20 + 0.12i;
     -0.91 - 2.06i, -0.66 - 0.42i,  0.63 + 0.17i, ...
                    -0.98 + 0.36i, -0.17 + 0.46i, -0.07 - 1.23i;
     -0.05 - 0.41i, -0.81 - 0.56i, -1.11 - 0.60i, ...
                     0.22 + 0.20i,  1.47 - 1.59i,  0.26 - 0.26i];
m = nag_int(size(a,1));
n = nag_int(size(a,2));
b = complex(ones(m,1));

jobz = 'Singular vector parts of U and VT';
[~, s, u, vt, info] = f08kr( ...
                             jobz, a);

disp('Singular values of A');
disp(s');

% Use SVD to compute minimum-norm solution: VS^(-1)U'b
y = u'*b;
y = y./s;
x = vt'*y;

disp('Minimum-norm solution:');
disp(x);
disp('Norm of Solution:');
disp(norm(x));
disp('Norm of Residual:');
fprintf('%11.1e\n',norm(b - a*x));
```

9.2 Program Results

```
     f08kr example results

Singular values of A
    3.9994    3.0003    1.9944    0.9995

Minimum-norm solution:
  -0.4024 + 0.3777i
  -0.2272 + 0.3626i
   0.1704 - 0.1532i
   0.2125 + 0.0781i
   0.2041 + 0.2236i
   0.2766 - 0.1517i
```

```
Norm of Solution:
    0.8846

Norm of Residual:
    1.1e-15
```
